Nanoscale superconductivity: Smaller is different and more

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Pedro Ribeiro
Dresden

PRB, 86, 064526 (2012)

PRL 108, 097004 (2012)

PRB 84,104525 (2011)
Editor‘s Suggestion

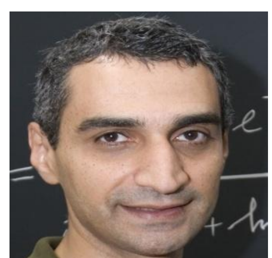
PRB 83, 014510 (2011)

Nature Materials 9, 550 (2010)

Sangita Bose
Bombay

Santos & Way
Santa Barbara

Altshuler
Columbia



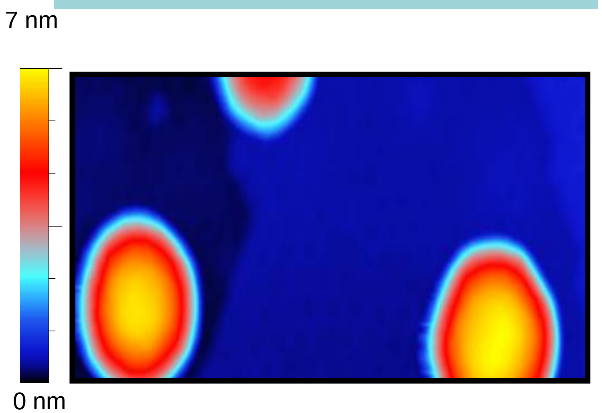
Yuzbashyan
Rutgers

Richter & Urbina
Regensburg

Klaus Kern
Stuttgart

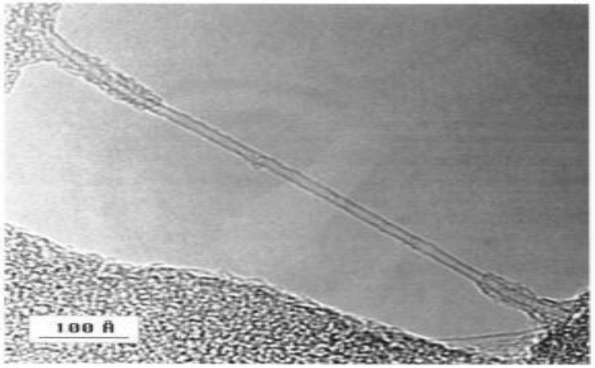
Single grains

$R \ll \xi$



Nanowires

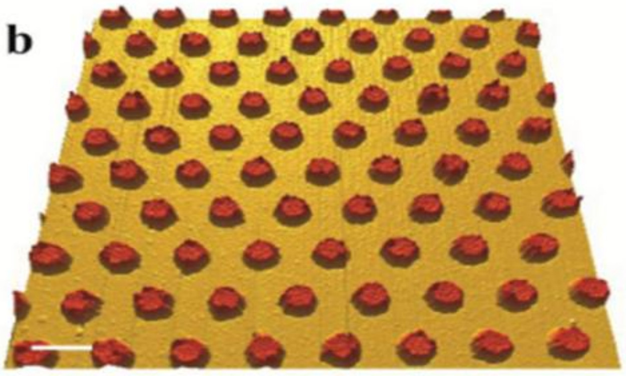
$R \ll \xi$



What?

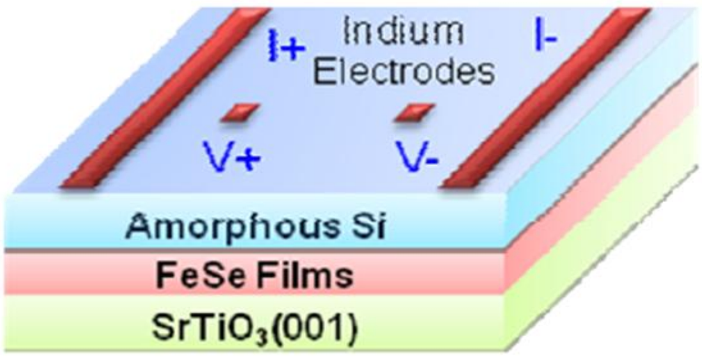
JJ Arrays

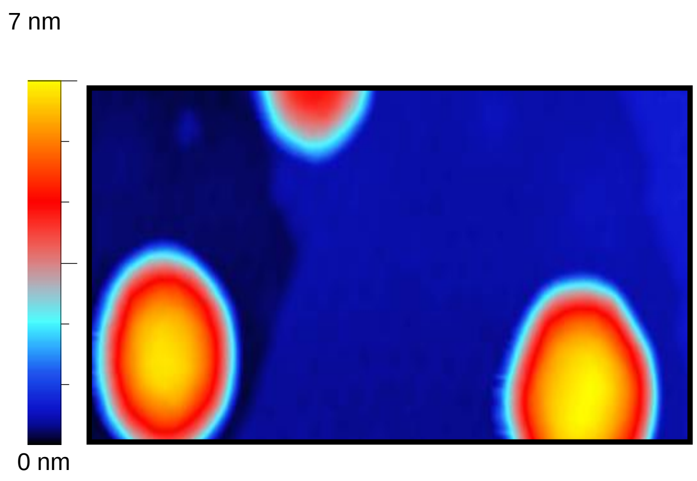
$R, l \ll \xi$



Thin Films

$L_z \ll \xi$

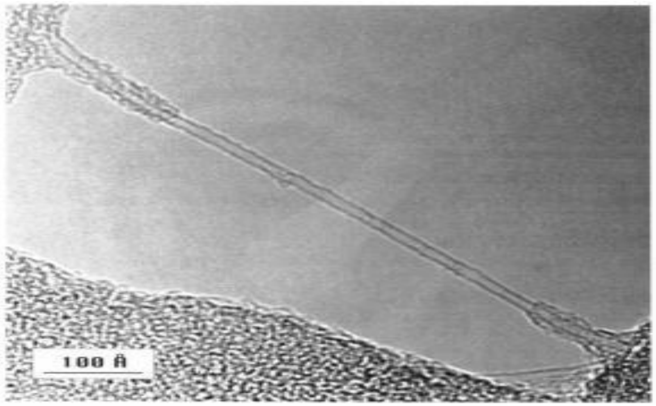




Why?

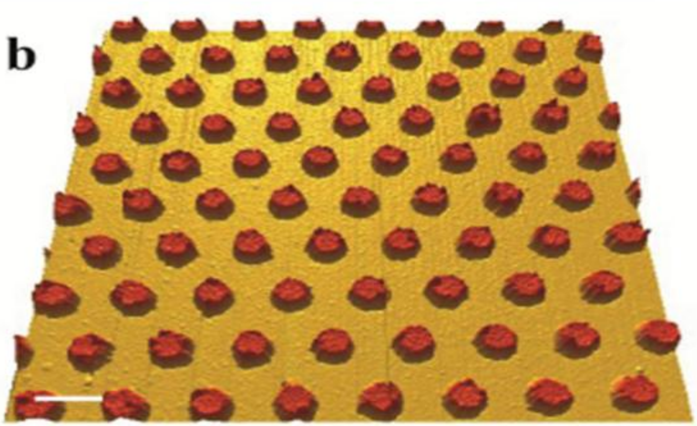
Mesoscopic + SC

Beauty of quantum coherence



Nanocircuits

Where is the limit?



Enhancement of Tc?

Despite Mermin-Wegner theorem?

Enhancement?

How to enhance SC substantially?

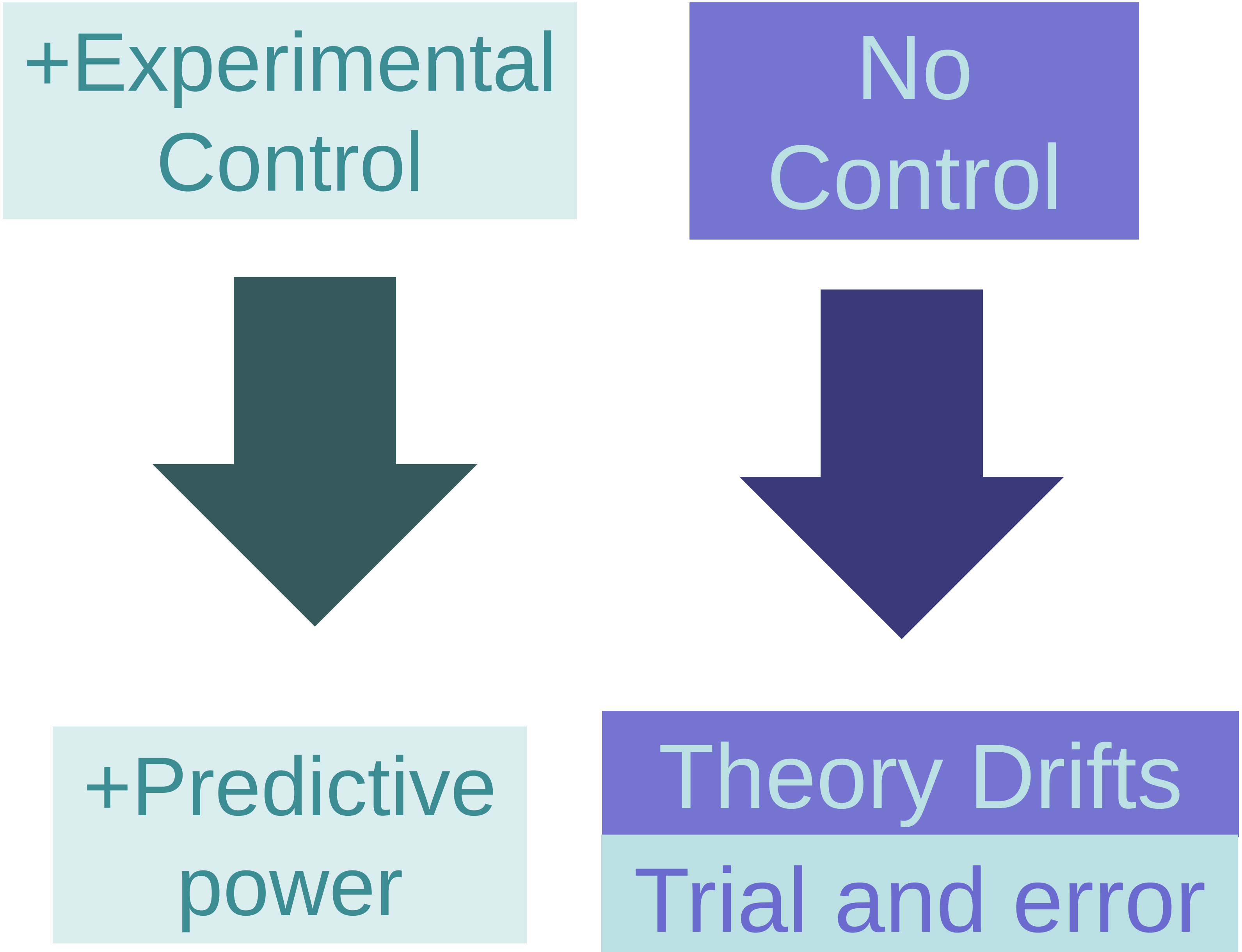
with **control**

Mechanism of SC in cuprates?

?

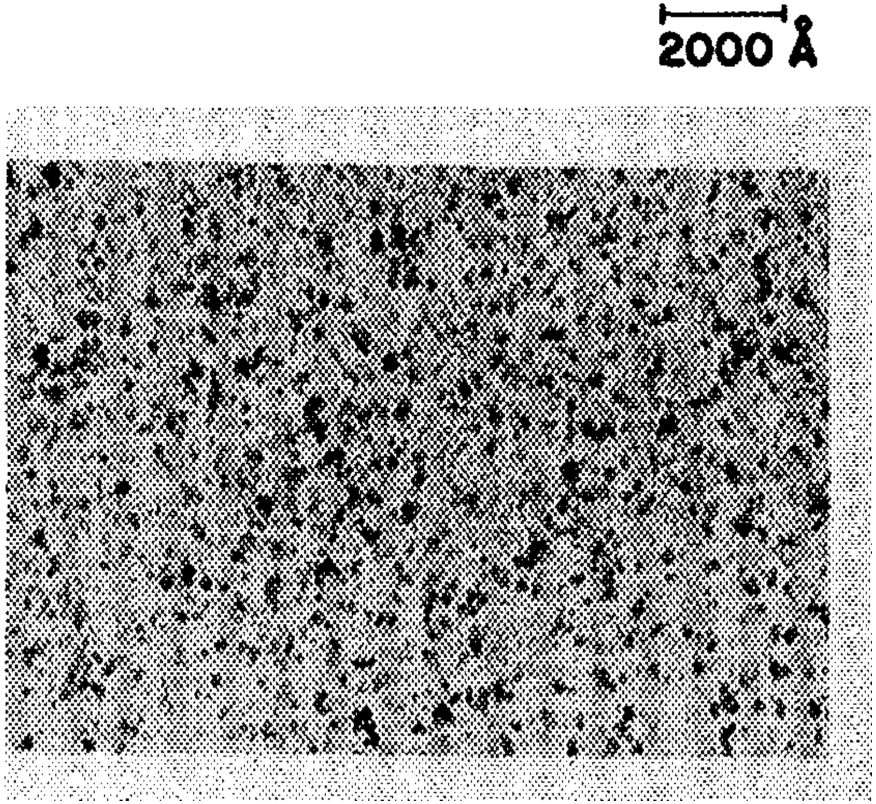
$$10^6 Question

$10 Question


+Experimental Control
No Control
+Predictive power
Theory Drifts
Trial and error

Thin Films? JJ array?

Metal	T_c (°K)	T_c/T_{c0}	d (Å)	ρ_0
Al	3.0	2.6	40	0.19
Ga	7.2	6.5	···	0.20
Sn	4.1	1.1	110	0.31
In	3.7	1.1	110	0.36
Pb	7.2	1.0	···	0.53



Abeles, Cohen, Cullen, Phys. Rev. Lett., 17, 632 (1966)

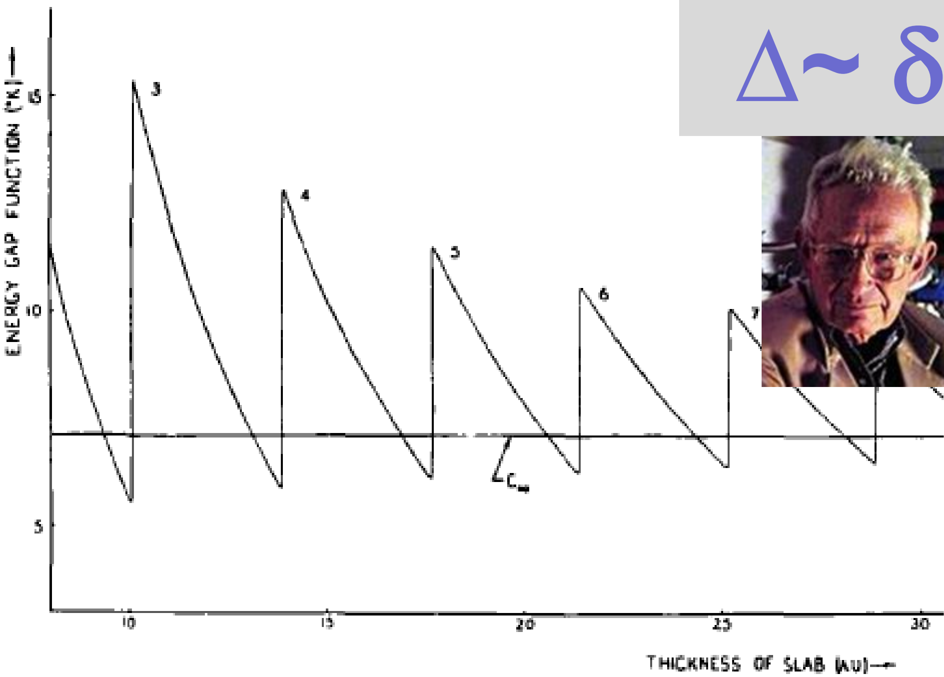
Crow, Parks, Douglass, Jensen, Giaver, Zeller....

A.M. Goldman, Dynes, Tinkham…

Thin Films

Single grains

$\Delta \sim \delta$?



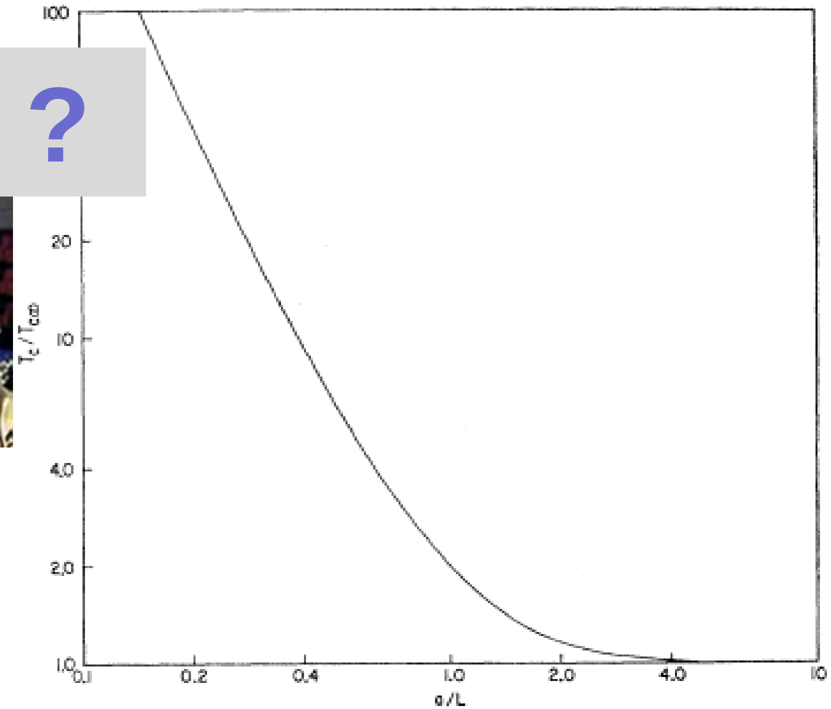


FIG. 1. $(T_c/T_{c\infty})$ versus (a/L) (see Ref. 17).

Shape Resonances

Blatt, Thompson
Phys. Lett. 5, 6 (1963)

Shell Effects

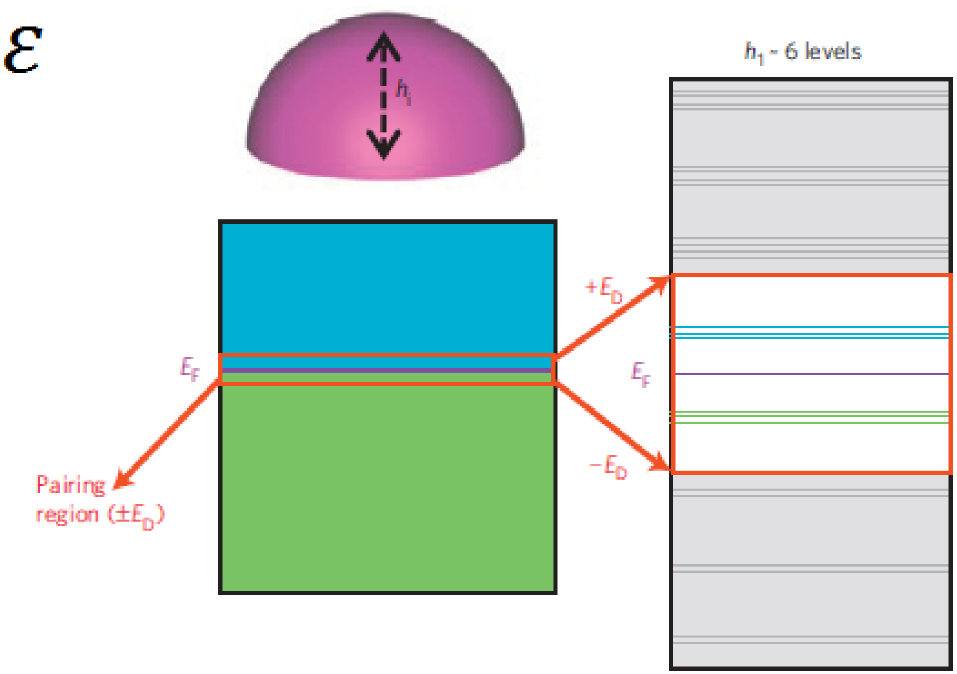
Parmenter, Phys. Rev. 166, 392 (1967)

BCS superconductivity

Finite size effects

$$\frac{2}{g} = \int_{-E_D}^{E_D} \frac{\nu(\varepsilon)}{\sqrt{\Delta^2 + \varepsilon^2}} d\varepsilon$$

$$\nu(\varepsilon) = \sum_i c_i \delta(\varepsilon - \varepsilon_i)$$



$V \rightarrow \infty$

$\Delta \sim \varepsilon_D e^{-1/\lambda}$

V finite

Δ=?

Thinner

Smoother

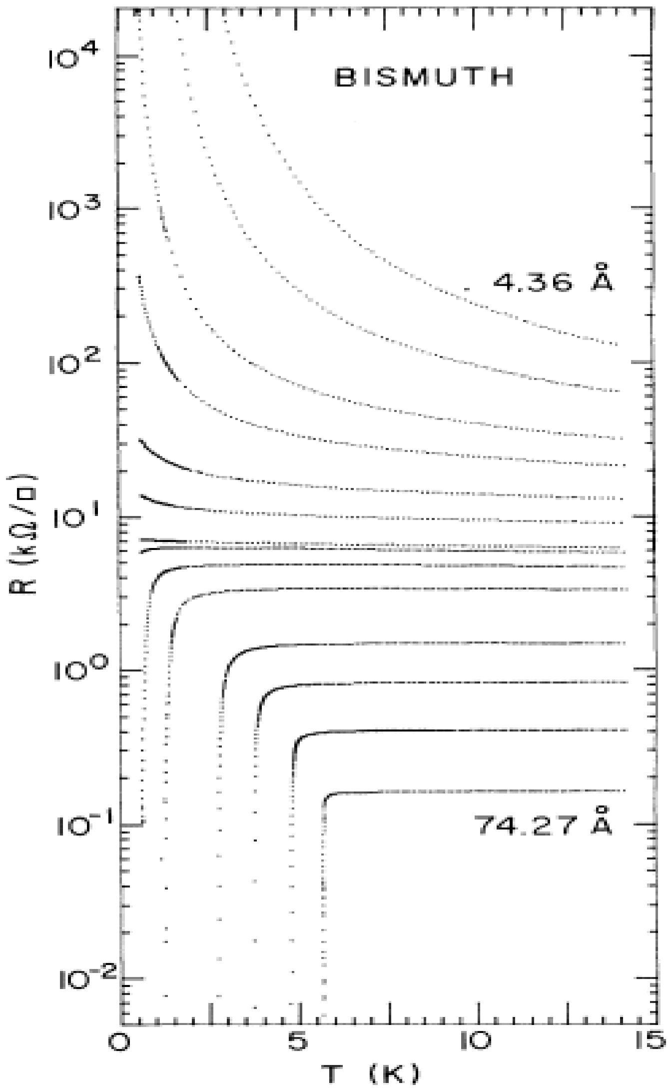
Disordered

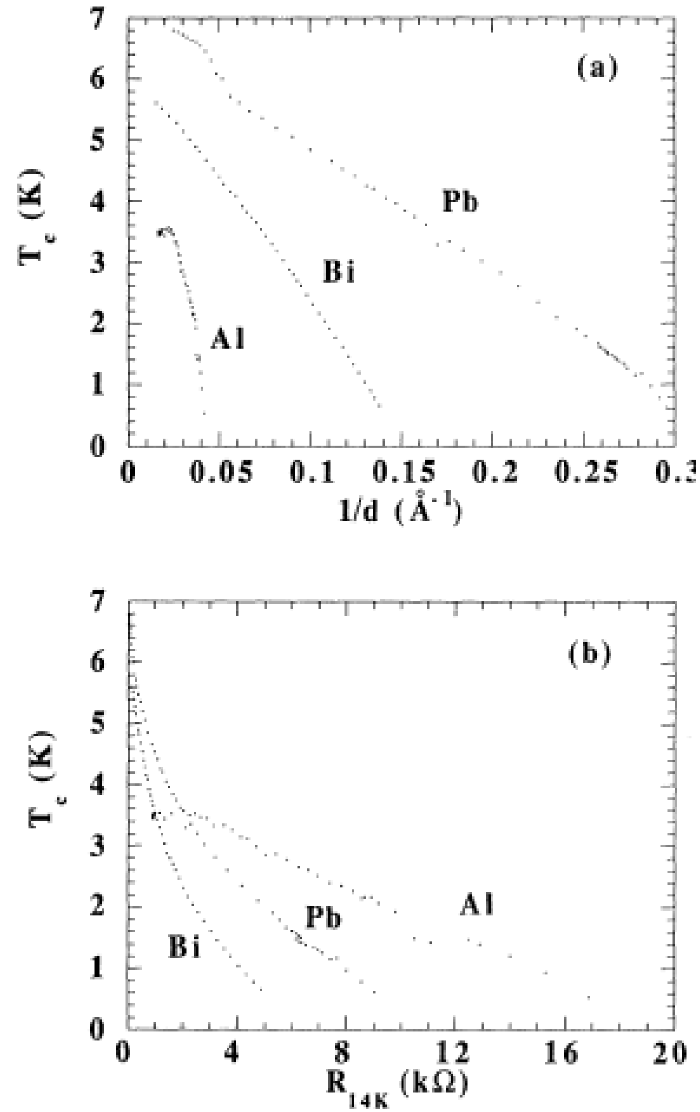
BKT

Transition

$R_N > R_q$

Vortices unbinding

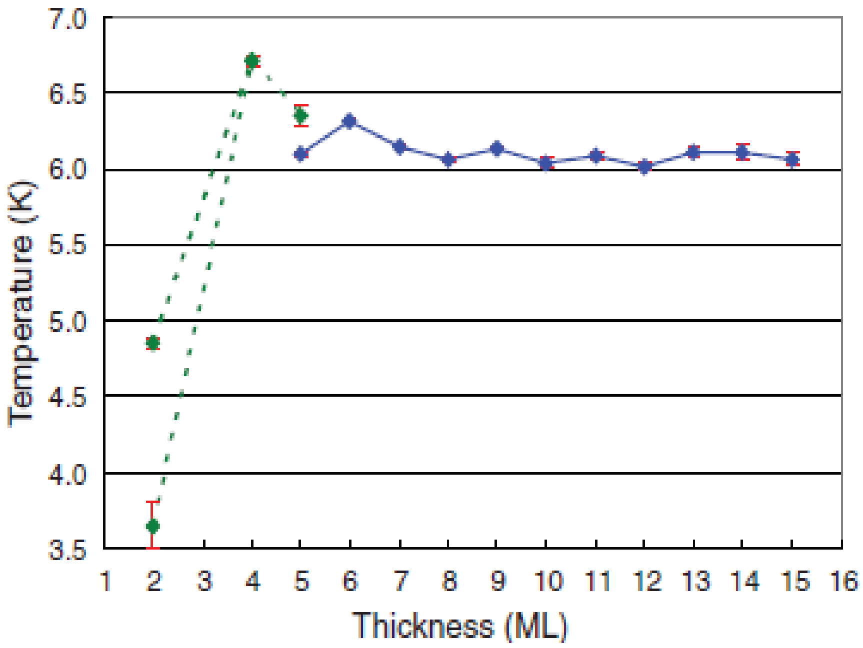




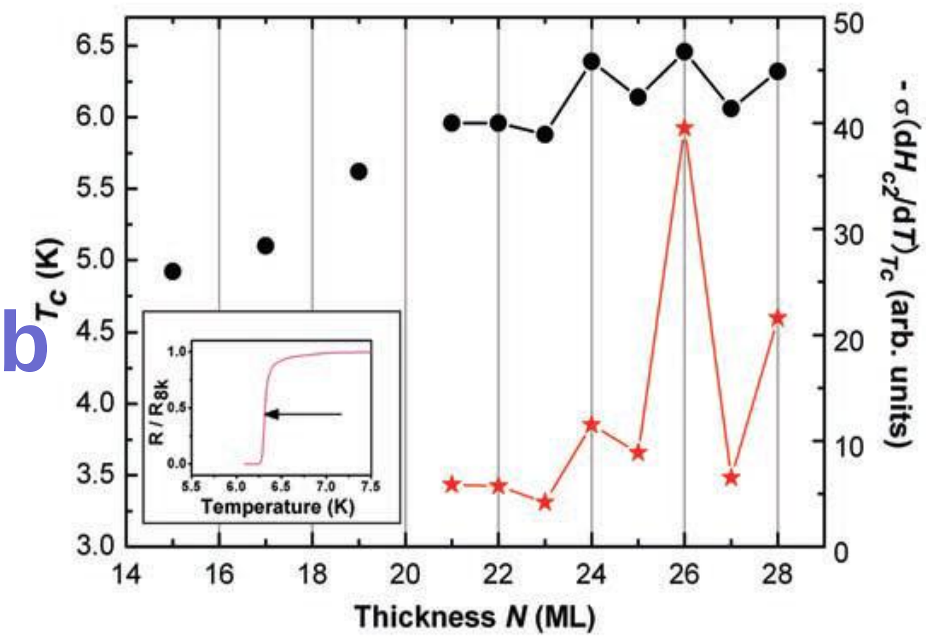
A.M. Goldman et al.

PRL 62 2180 (1989)
PRB 47 5931 (1993)

Recent Atomic scale **control**



Pb

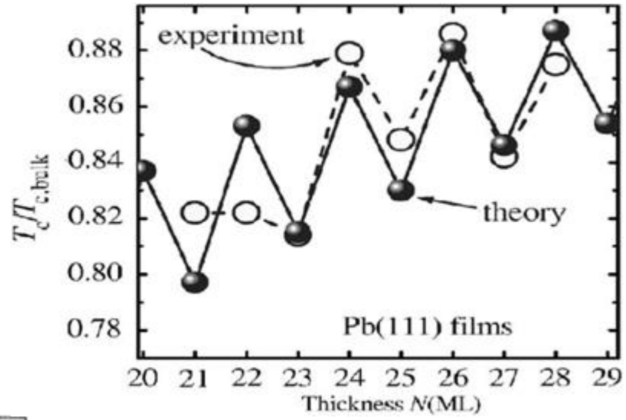


Shih et al., Science 324, 1314 (2009)

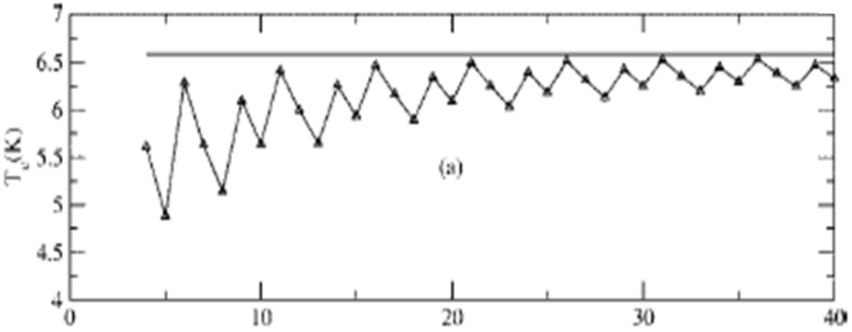
Xue et al., Science 306, 1915 (2004)

Xue et al., Nat Phys, 6 (2010),104.

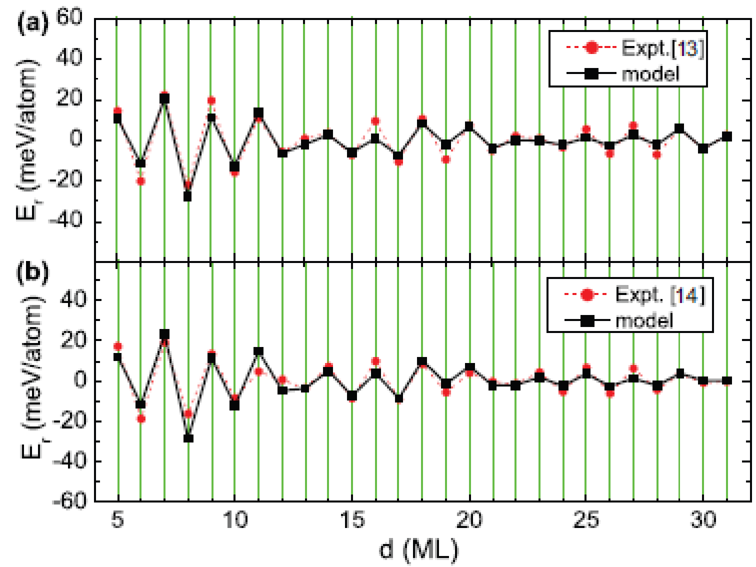
Quantum size effects



PRB 75 014519 (2007)



PRB 74 132504 (2006)

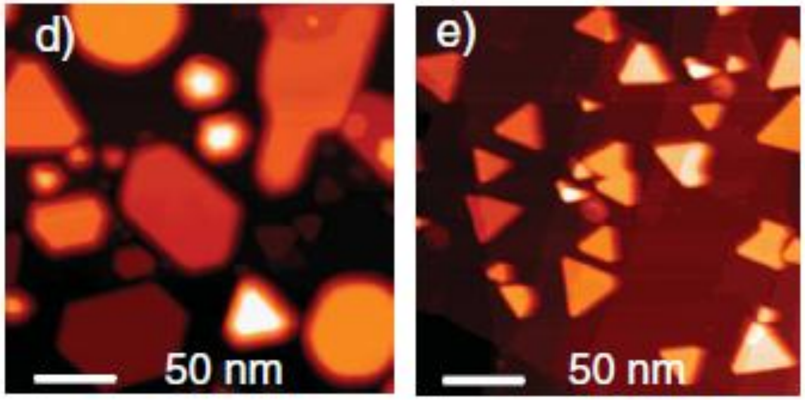


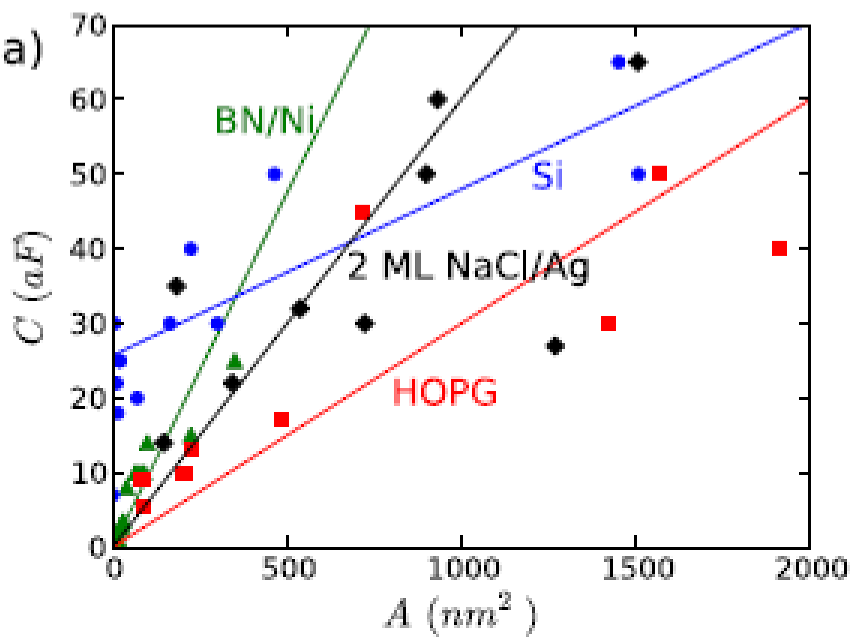
Stress, substrate

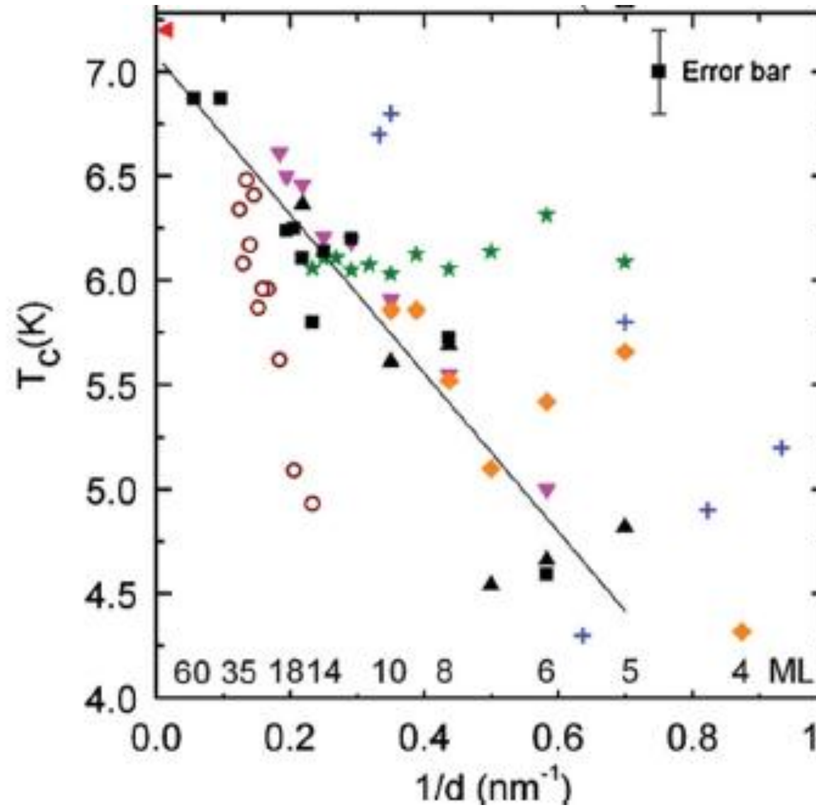
Xue, Liu et al.
arxiv:1208.6054

Islands

Pb/BN/Ni(111) Pb/NaCl/Ag(111)







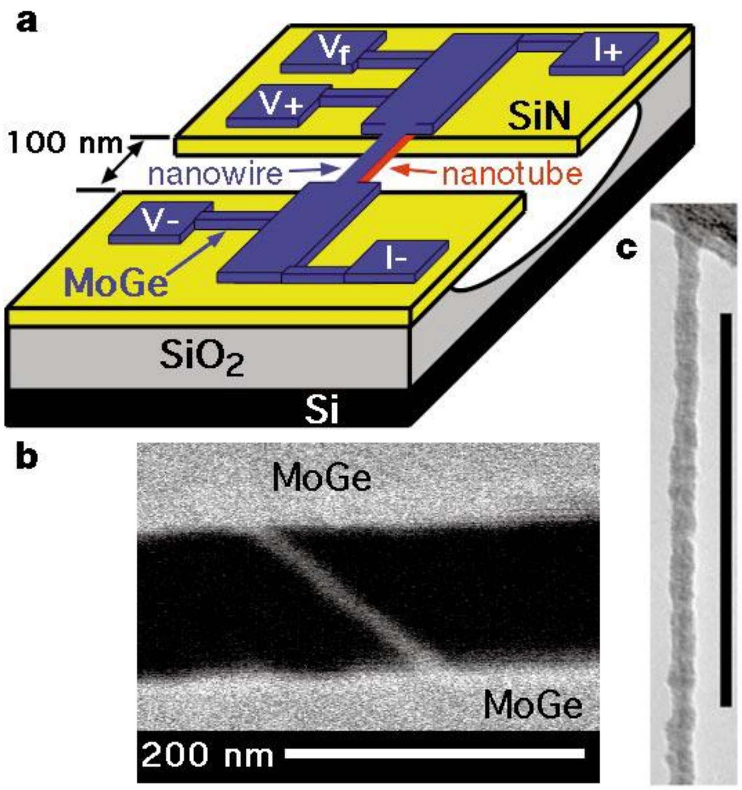
Schneider, et al.,
PRL 102, 207002 (2009)

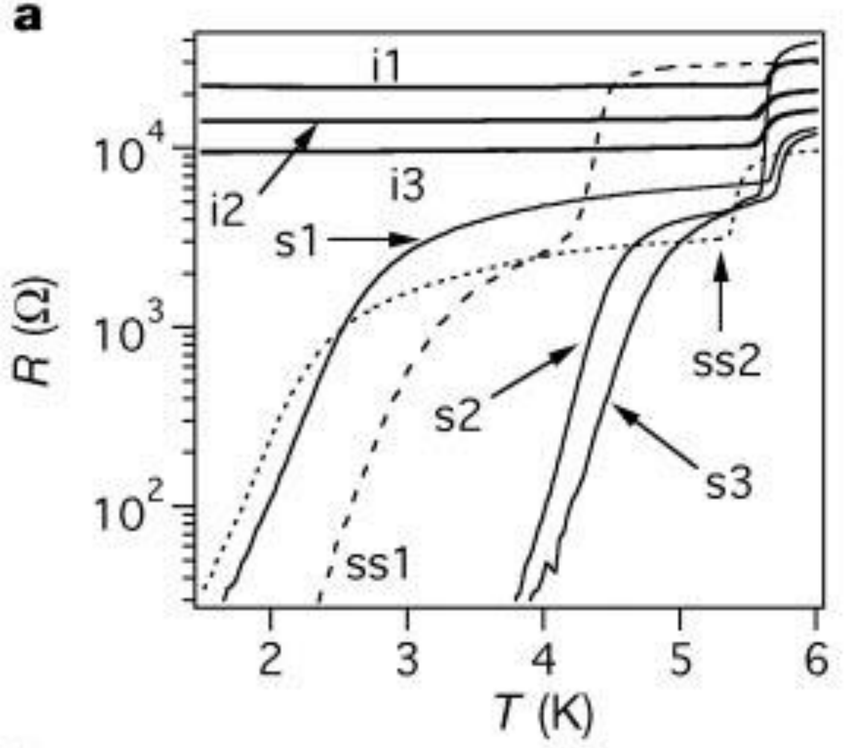
PRL 108, 126802 (2012)

Hasegawa, et al.

Phys. Rev. Lett. 101, 167001 (2008)

Nanowires R <<ξ





Superconductor
Insulator
transition

Tinkham et al.
Nature 404, 971 (1990)

$$|\Delta(\mathrm{r},\mathrm{t})|e^{i\theta(r,t)}$$

Fluctuation

$\Delta(r_0, t_0) \approx 0$

Phase-slips

$\theta \approx 0 \to 2\pi$

Finite Resistence

$R \propto e^{-S_{inst}}$

Thermal

Langer & Ambegaokar, PR. 164, 498 (1967).
McCumber & Halperin PRB 1, 1054 (1970).

Quantum

Zaikin, A. D., Golubev, et al, PRL 78, 1552 (1997).

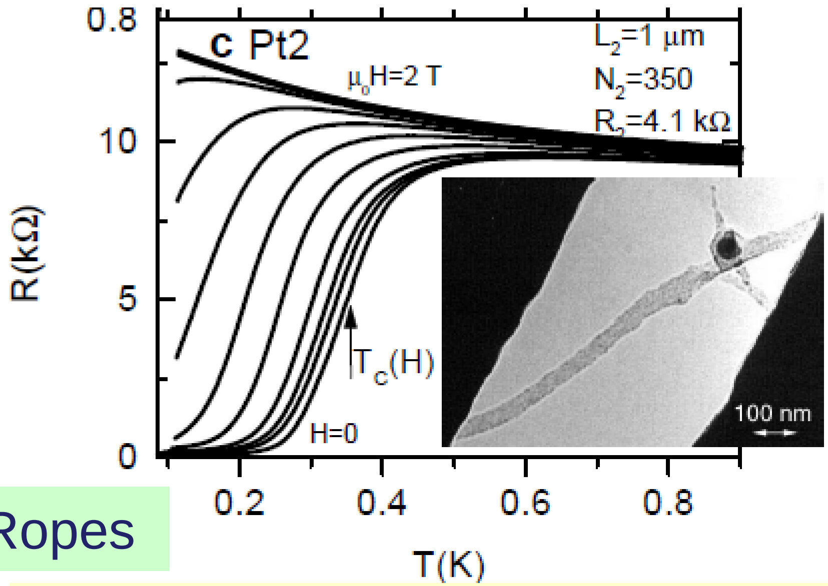
Instantons

Coulomb-Gas

BKT transition

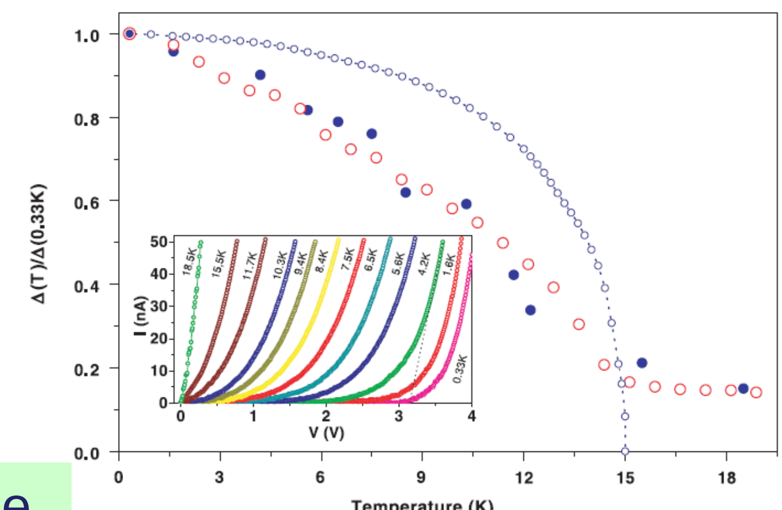
Quantitative?

Carbon nanotubes



Ropes

Phy. Rev. Lett. 86, 2416 (2001)



Single

Science 292, 2462 (2001)

Fluctuations

High Tc?

Phase Slips

Lehtinen, PRB 85 094508 (2012)

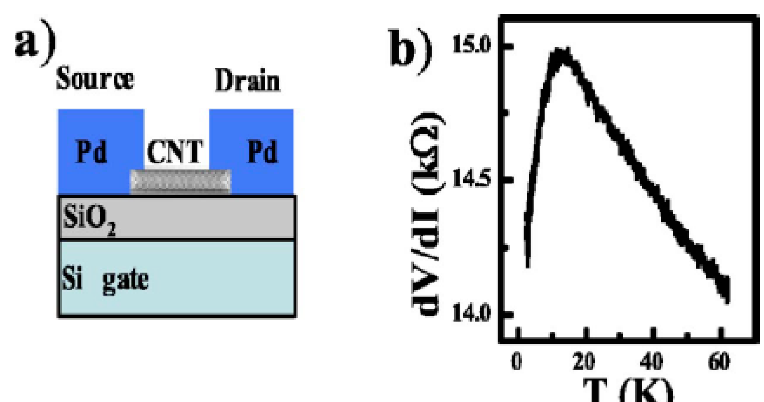
How to suppress fluctuations?

Dissipation

PRB 80, 214515 (2009)

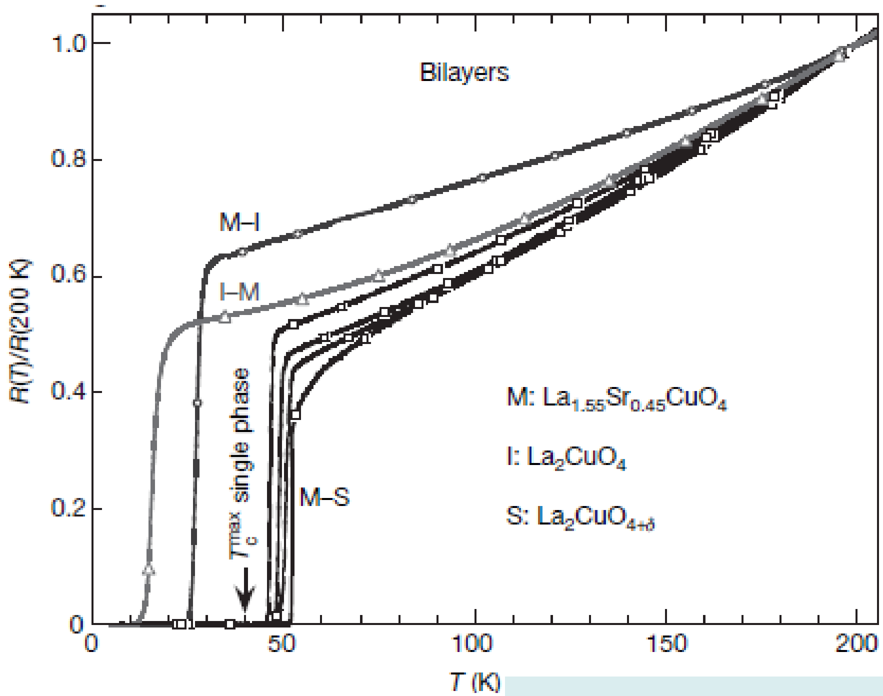
Electric field effect

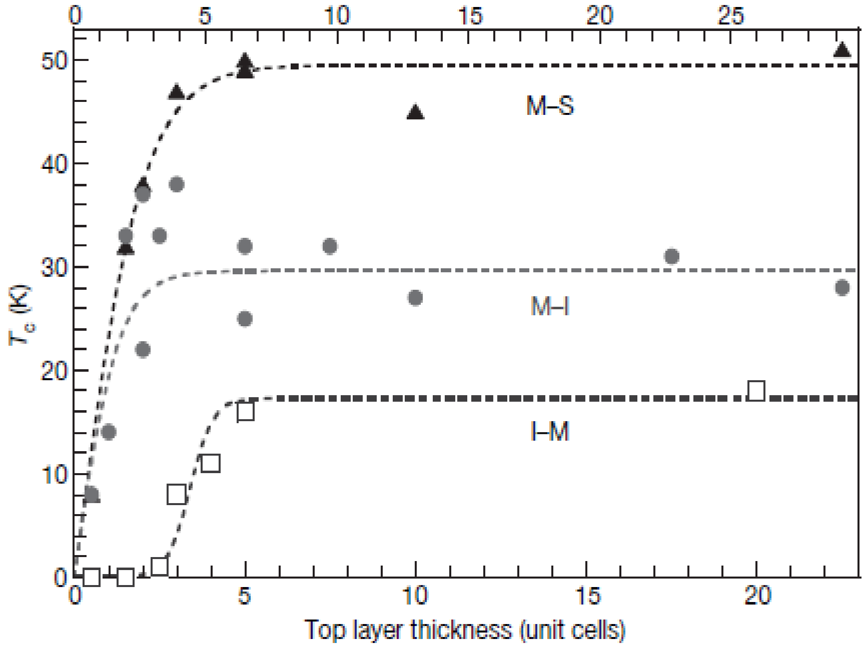
Yang,PRB 74 155414 (2006)



Is enhancement of superconductivity possible?

Cuprates high T_c Heterostructures



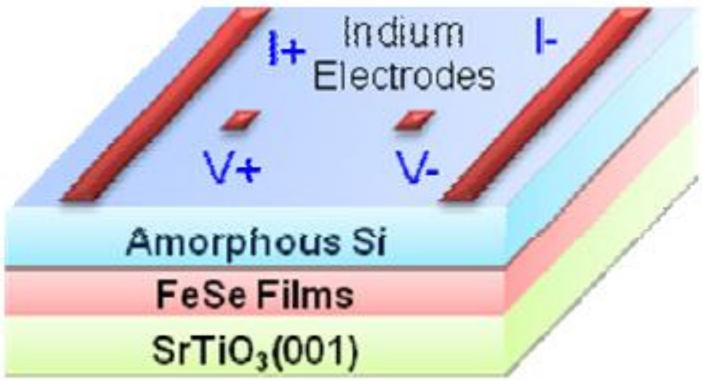


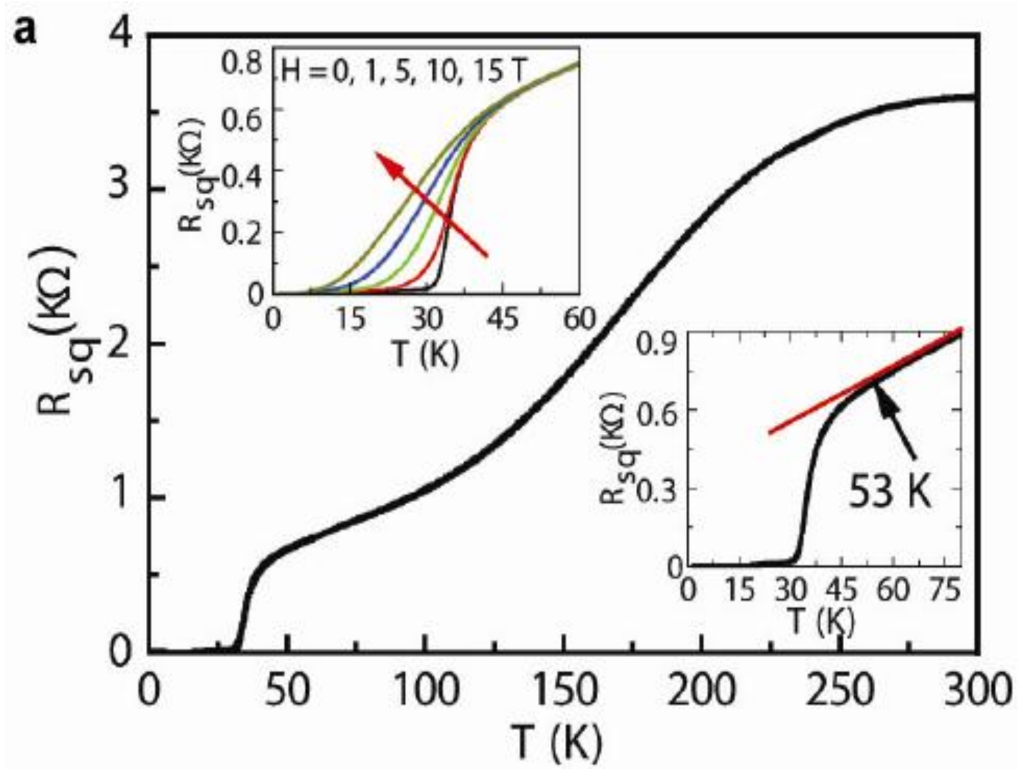
Bozovic et al., Nature 455, 782 (2008)

Higher T_c!!

Intrinsic inhomogeneities

Iron Pnictides Heterostructures

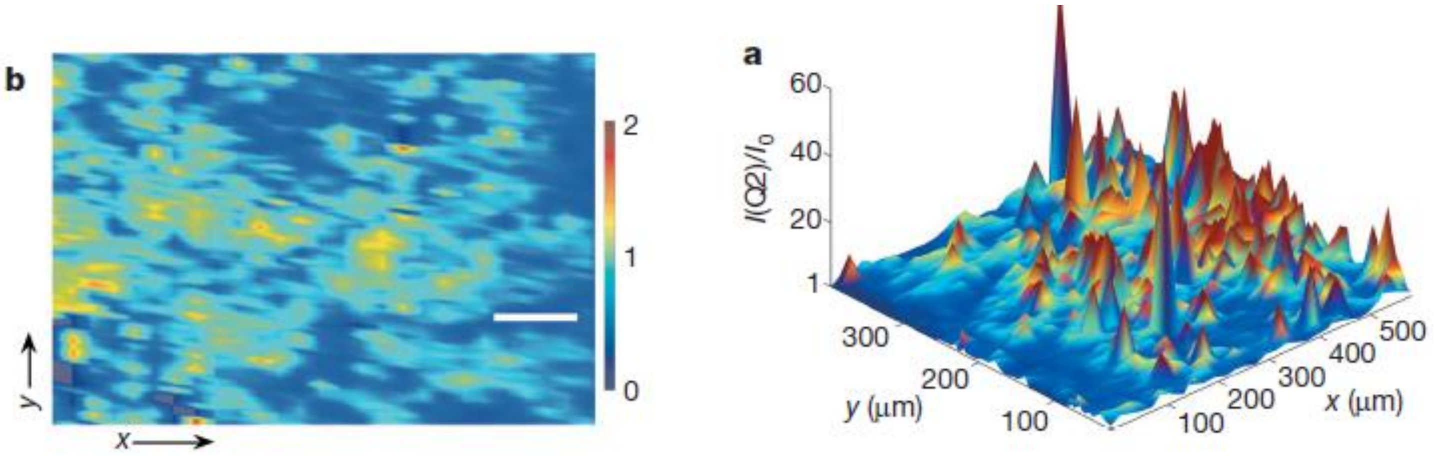




Xue et al.: Arxiv: 12015694

Enhancement of T_c by disorder

Fractal distributions of dopants enhances SC in cuprates

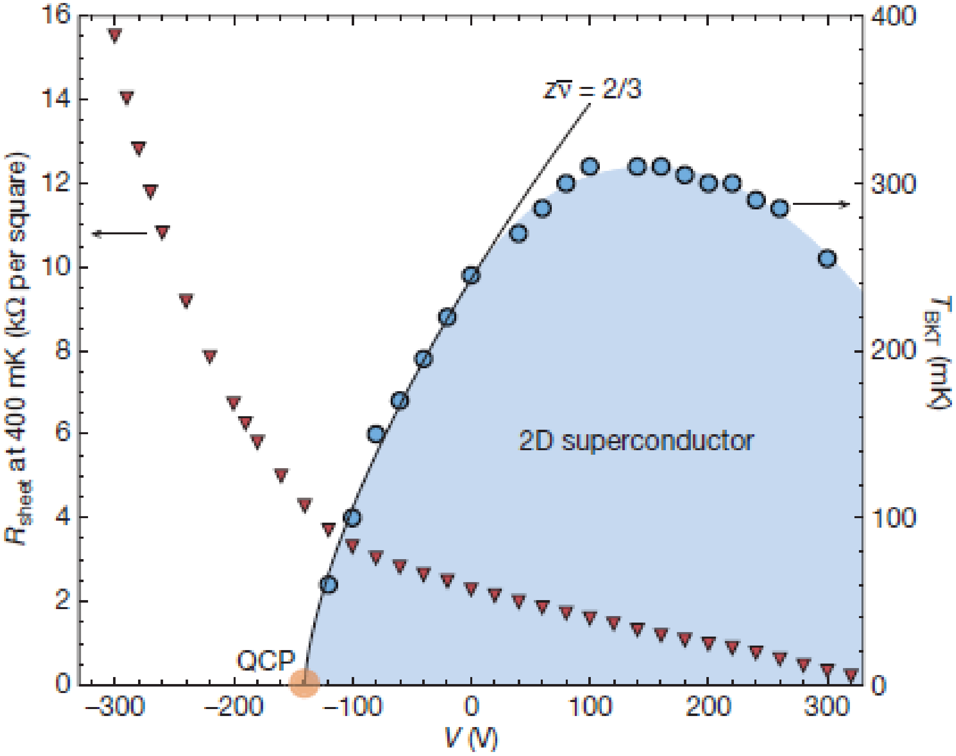


Bianconi, et al., Nature 466, 841 (2010)

Inhomogeneities ⟷ Higher T_c

PRL 108, 017002 (2012)

$LaAlO_3$ /$SrTiO_3$ Heterostructures



Triscone, Nature 456 624 (2008)

Lesueur, arXiv:1112.2633

PRL 104, 126803 (2010)
PRB 85 020457 (2012)

Control &Tunability

Spin-Orbit

Disorder

Magnetism

E Field effect

Relevance

Localization

Exotic Quantum Matter

Topology

Enhancement, yes

Origin?

Grains

$\Delta \sim \delta$

Supercon ductivity?

1959

Yes, superconductivity

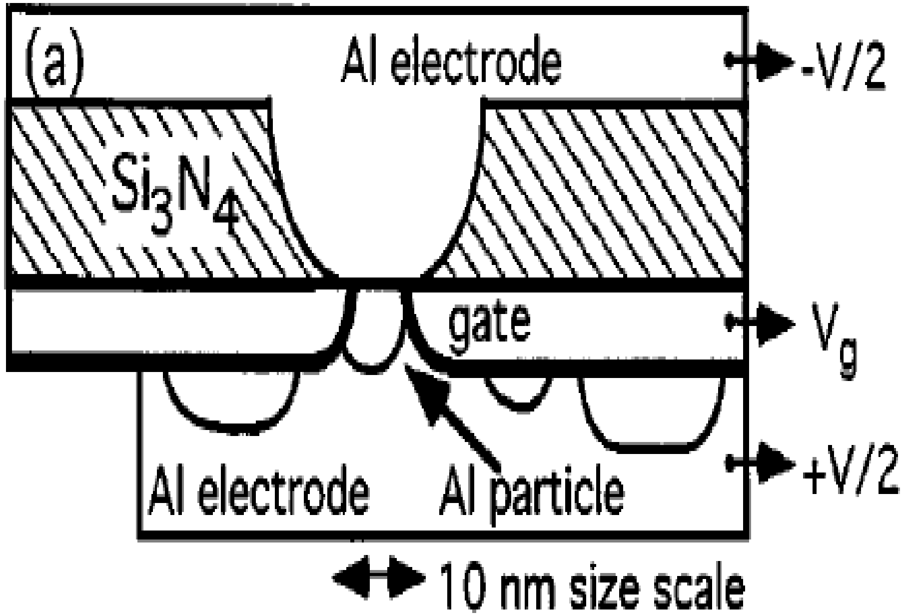
B closes gap

Odd-even effects

Isolated grain?

Ralph, Black,Tinkham, Superconductivity in Single Metal Particles

***PRL* 74, 3241-3244 (1995).**



Theoretical response

T = 0
Ultrasmall grains
$\delta / \Delta_0 > 1$

von Delft, Braun, Larkin, Sierra, Dukelsky, Yuzbashyan, Matveev, Smith, Ambegaokar

Exact diagonalization, RPA, Path Integral, Montecarlo......

Richardson

It's exact. I did it 20 years ago

BCS fine until $\delta / \Delta_0 \sim 1/2$

BCS sharp transition

Richardson no transition

J. von Delft et al., Phys. Rep., 345, 61 (2001)

$\Delta \gg \delta$

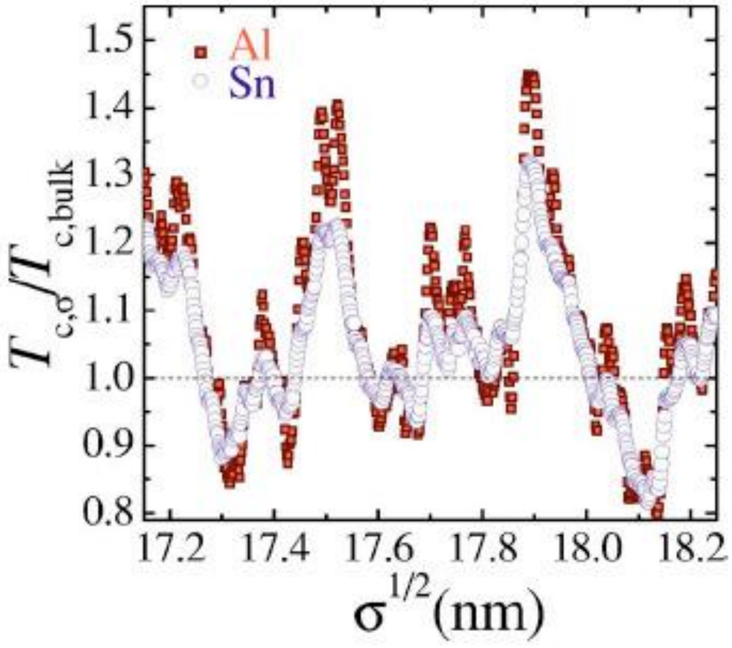
Heiselberg (2002): harmonic potentials, cold atom

Devreese (2006): Richardson equations in a box

Kresin, Boyaci, Ovchinnikov (2007) : Spherical grain, high T_c

Olofsson (2008): Estimation of fluctuations in BCS

Peeters, Shanenko, Croitoru, (2005-): BCS, BdG in a wire, cylinder..

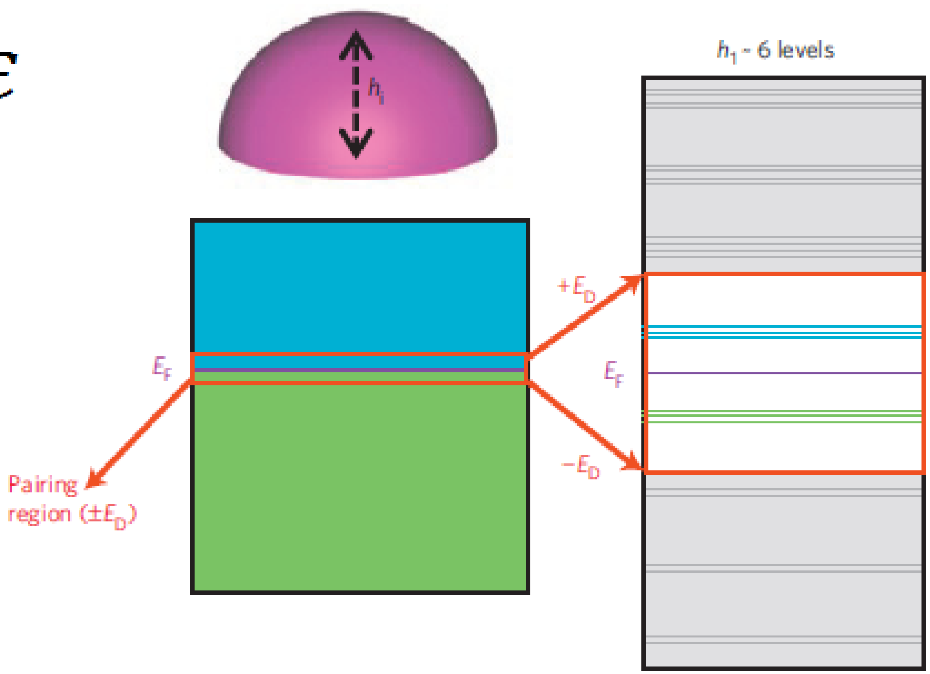


Enhancement of SC is possible!

BCS superconductivity	Finite size effects

$$\frac{2}{g} = \int_{-E_D}^{E_D} \frac{\nu(\varepsilon)}{\sqrt{\Delta^2 + \varepsilon^2}} d\varepsilon$$

$$\nu(\varepsilon) = \sum_i c_i \delta(\varepsilon - \varepsilon_i)$$



$V \to \infty$

$\Delta \sim \varepsilon_D e^{-1/\lambda}$

V finite

Δ=?

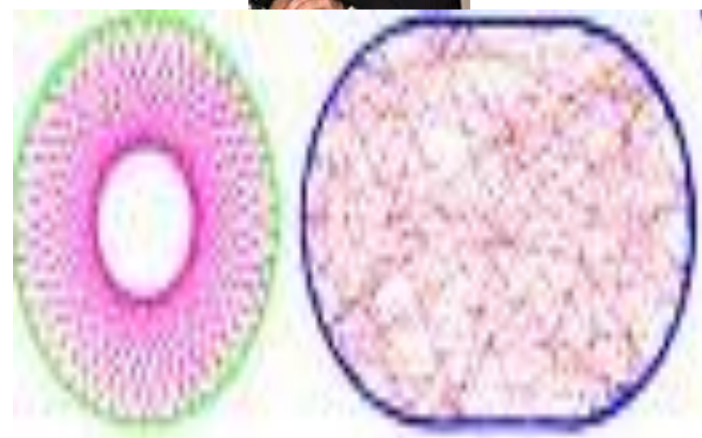
Chaotic grains?

Is it done already?

Go ahead!

This has not been done before

Analytical? $1/k_F L \ll 1$



Semiclassical techniques

Quantum observables in terms of classical quantities

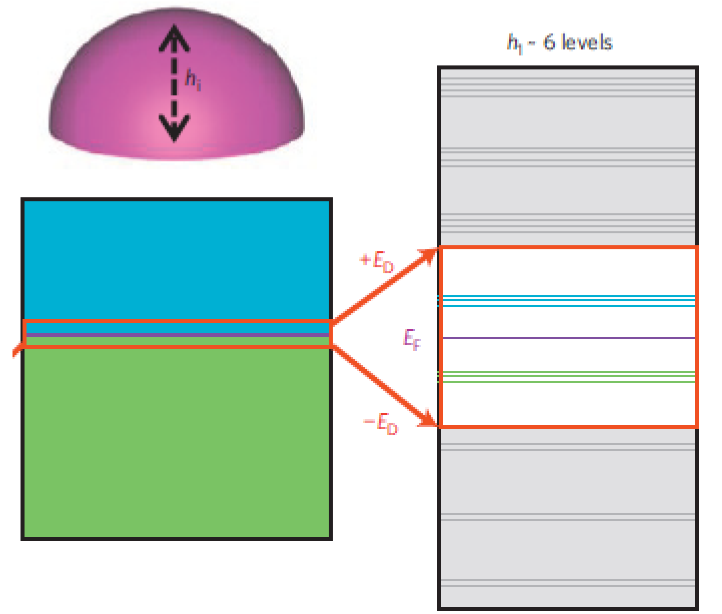
Berry, Gutzwiller, Balian, Bloch

$$\nu(\varepsilon) \Leftrightarrow L_p$$

$\Delta >> \delta$ L ~ 10nm

Bogouliobov de Gennes… difficult

BCS fine but..



$$H = \sum_{n\sigma} \epsilon_n c^{\dagger}_{n\sigma} c_{n\sigma} - \sum_{n.n'} I_{n,n'} c^{\dagger}_{n\uparrow} c^{\dagger}_{n\downarrow} c_{n'\downarrow} c_{n'\uparrow}$$

$$I(\epsilon_n, \epsilon_{n'}) = \lambda V \delta \int \psi_n^2(\vec{r}) \psi_{n'}^2(\vec{r}) d\vec{r}$$

$$\Delta(\epsilon) = \frac{1}{2} \int_{-\epsilon_D}^{\epsilon_D} \frac{\Delta(\epsilon') I(\epsilon, \epsilon')}{\sqrt{\epsilon'^2 + \Delta^2(\epsilon')}} \nu(\epsilon') d\epsilon'$$

Expansion in $1/k_F L$, δ/Δ_0

3d chaotic

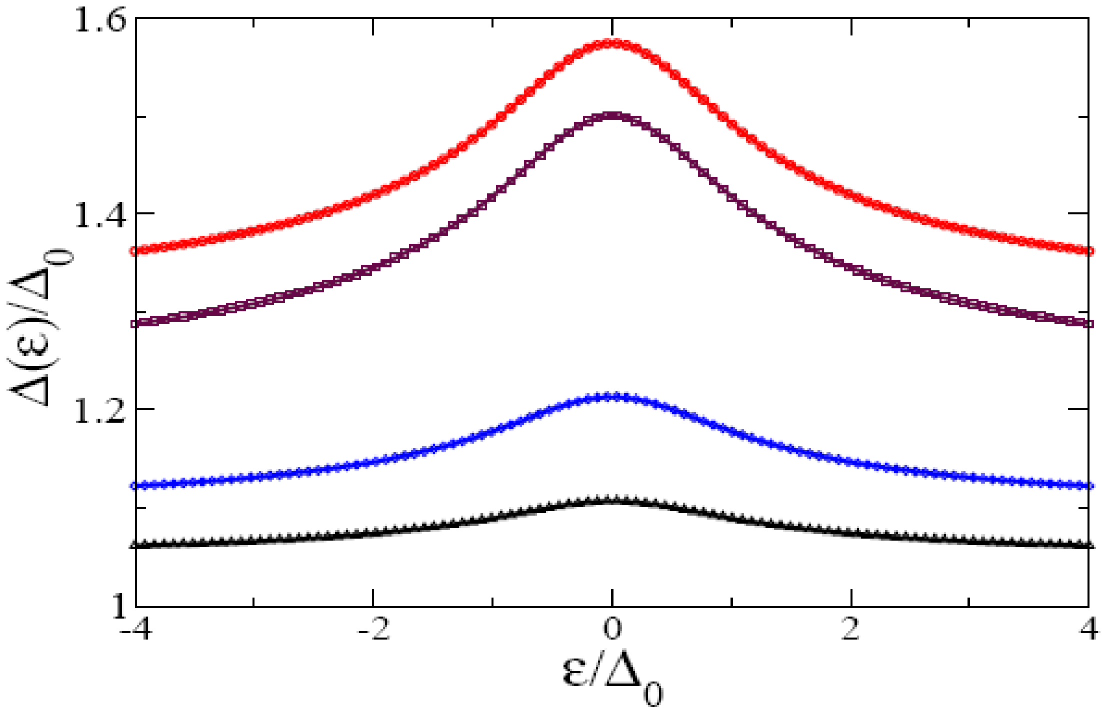
Al grain

k_F = 17.5 nm^{-1}

Δ_0 = 0.24mV

For L< 9nm leading correction comes from I

PRL 100, 187001 (2008)
PRB 83, 014510 (2011)



L = 6nm, Dirichlet, δ/Δ_0=0.67

L= 6nm, Neumann, $\delta/\Delta_{0,}$=0.67

L = 8nm, Dirichlet, δ/Δ_0=0.32

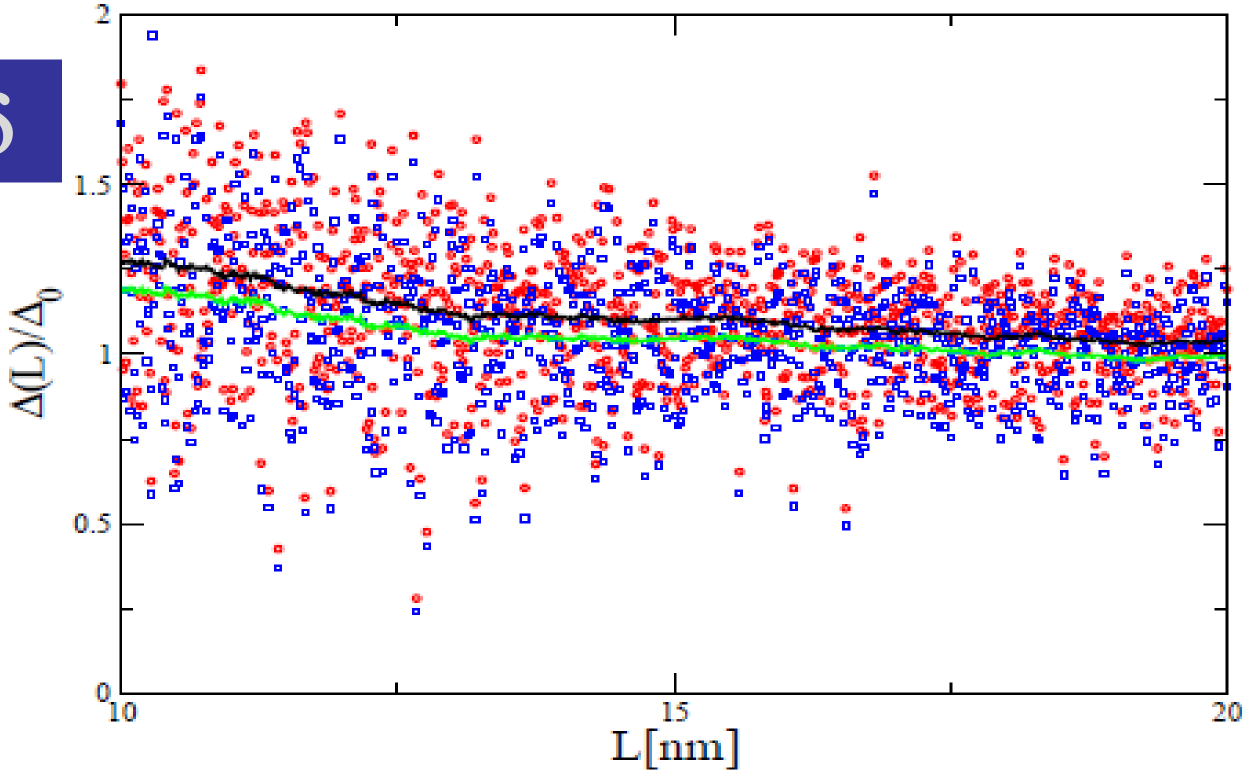
L = 10nm, Dirichlet, $\delta/\Delta_{0,}$= 0.08

Fluctuations
ξ > L

Symmetries

No fluctuations
ξ < L

$\Delta_0 >> \delta$



$$I(\epsilon_n, \epsilon_{n'}) = \lambda V \delta \int \psi_n^2(\vec{r}) \psi_{n'}^2(\vec{r}) d\vec{r}$$

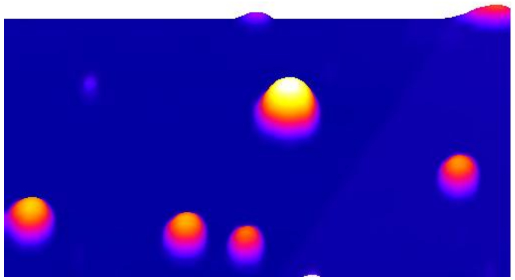
$$\nu(\varepsilon) = \sum_i c_i \delta(\varepsilon - \varepsilon_i)$$

Long range order?

Single, Isolated Sn and Pb grains

Kern

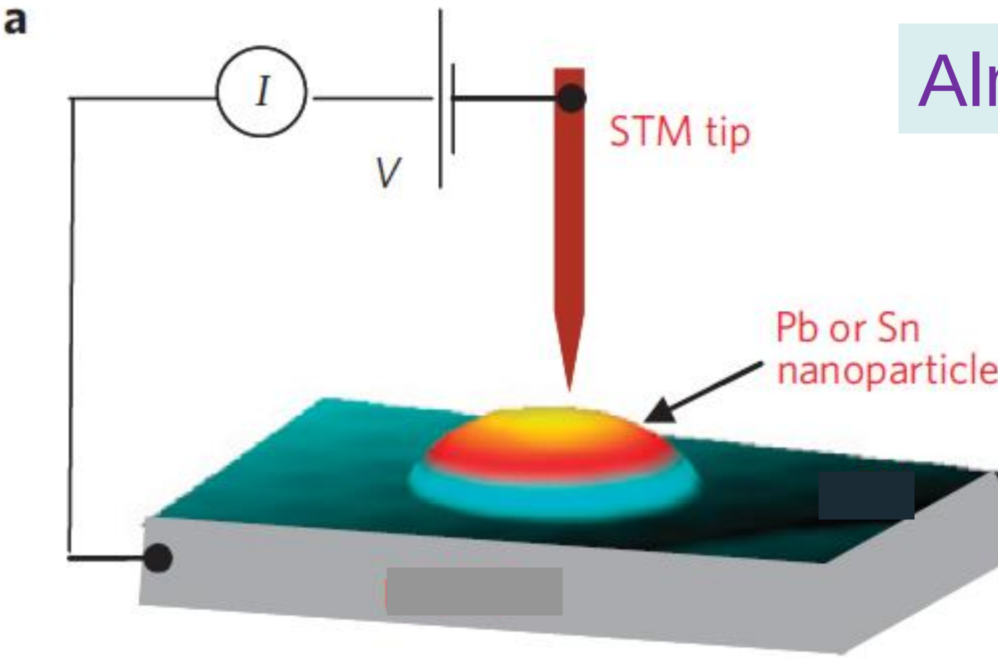
Bose



R ~ 4-30nm

B closes gap

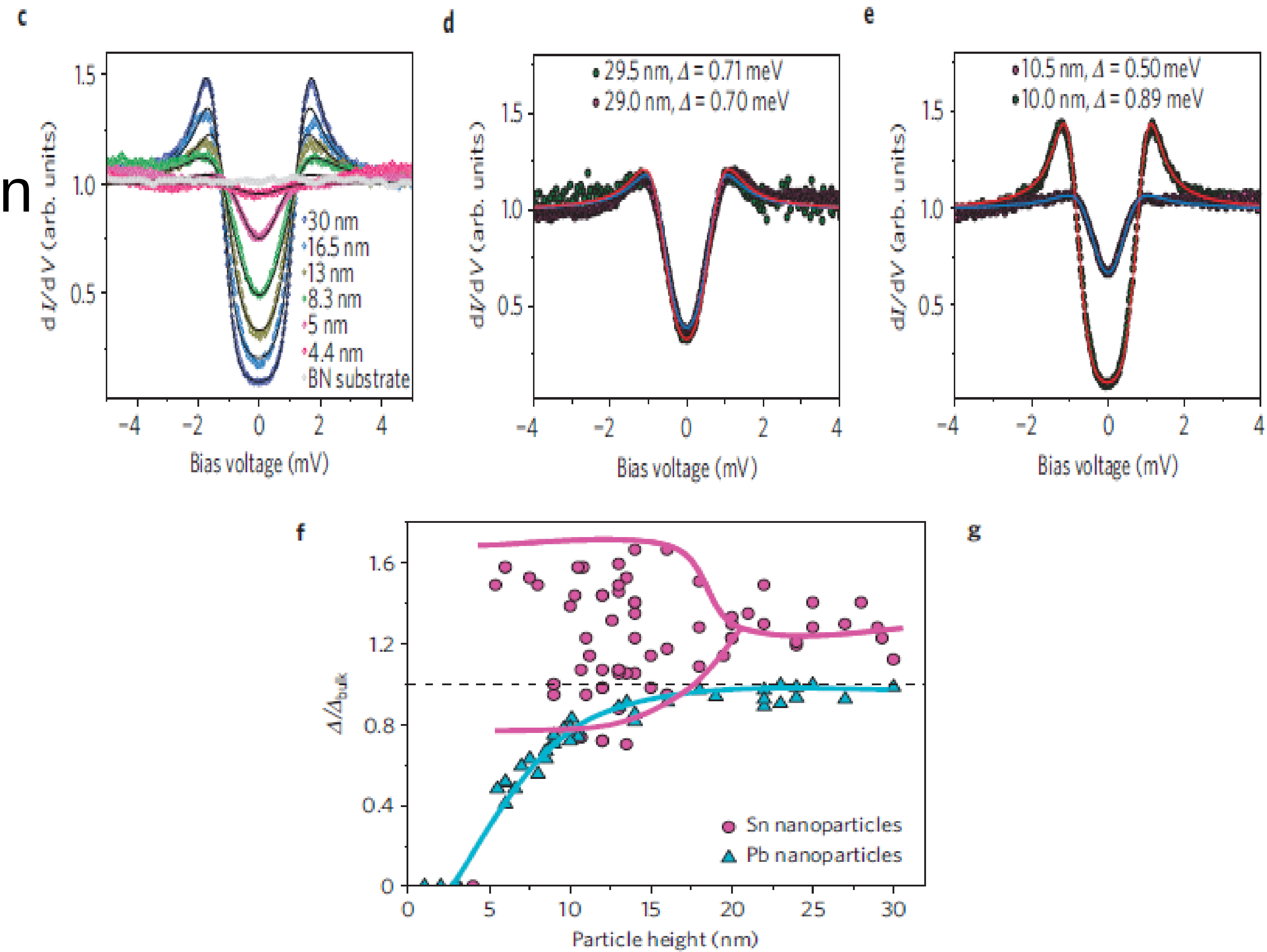
Almost hemispherical



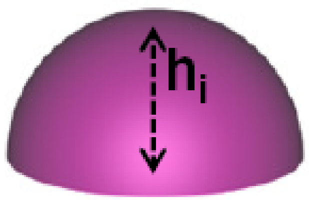
STM

Tunneling conductance

Sn

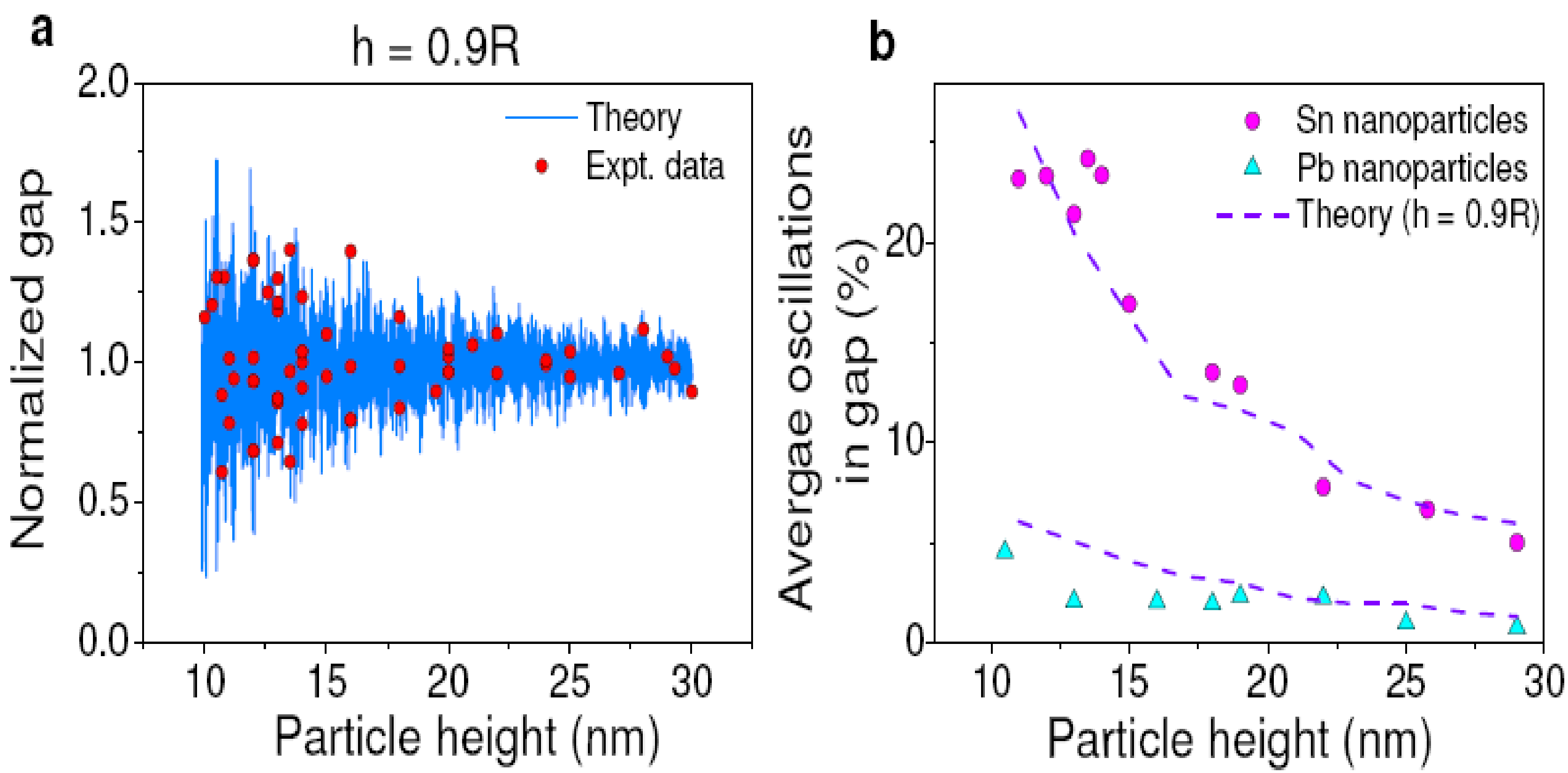


c
d
e
f
g
1.5
1.0
0.5
dI/dV (arb. units)
30 nm
16.5 nm
13 nm
8.3 nm
5 nm
4.4 nm
BN substrate
-4
-2
0
2
4
Bias voltage (mV)
29.5 nm, Δ = 0.71 meV
29.0 nm, Δ = 0.70 meV
10.5 nm, Δ = 0.50 meV
10.0 nm, Δ = 0.89 meV
Δ/Δbulk
1.6
1.2
0.8
0.4
0
5
10
15
20
25
30
Sn nanoparticles
Pb nanoparticles
Particle height (nm)



+

$$\Delta(\epsilon) = \frac{1}{2}\int_{-\epsilon_D}^{\epsilon_D} \frac{\Delta(\epsilon')I(\epsilon,\epsilon')}{\sqrt{\epsilon'^2+\Delta^2(\epsilon')}}\nu(\epsilon')d\epsilon'$$



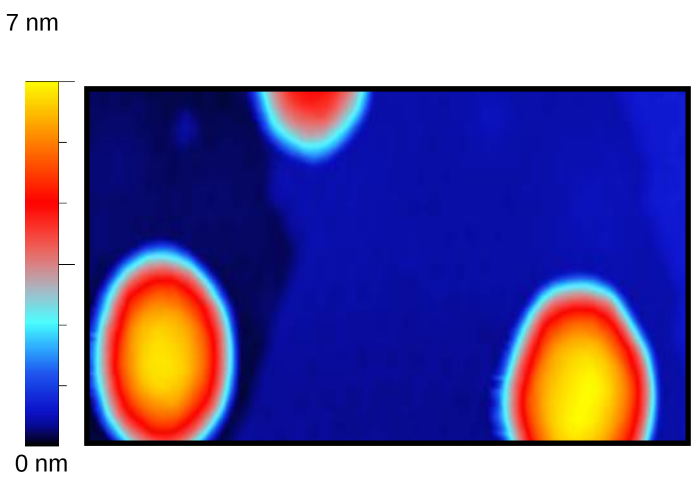
nature materials

LETTERS

PUBLISHED ONLINE: 30 MAY 2010 | DOI: 10.1038/NMAT2768

Observation of shell effects in superconducting nanoparticles of Sn

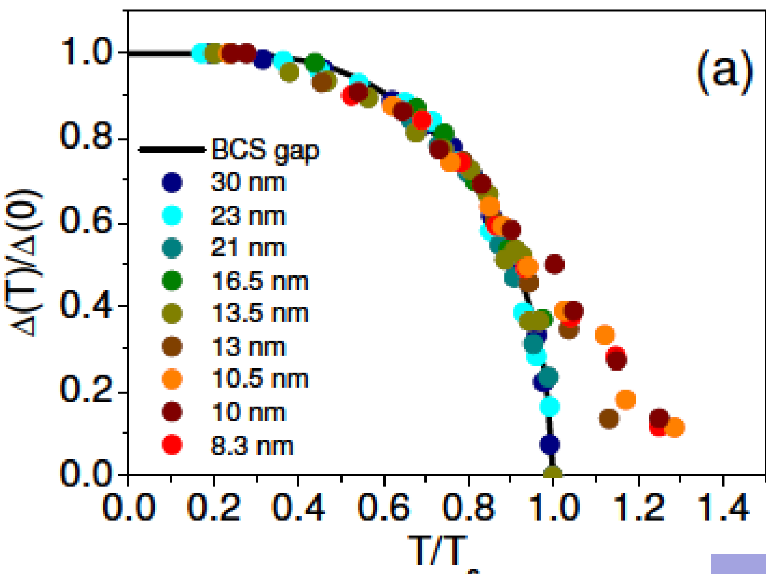
Sangita Bose[1]*, Antonio M. García-García[2]*, Miguel M. Ugeda[1,3], Juan D. Urbina[4], Christian H. Michaelis[1], Ivan Brihuega[1,3]* and Klaus Kern[1,5]



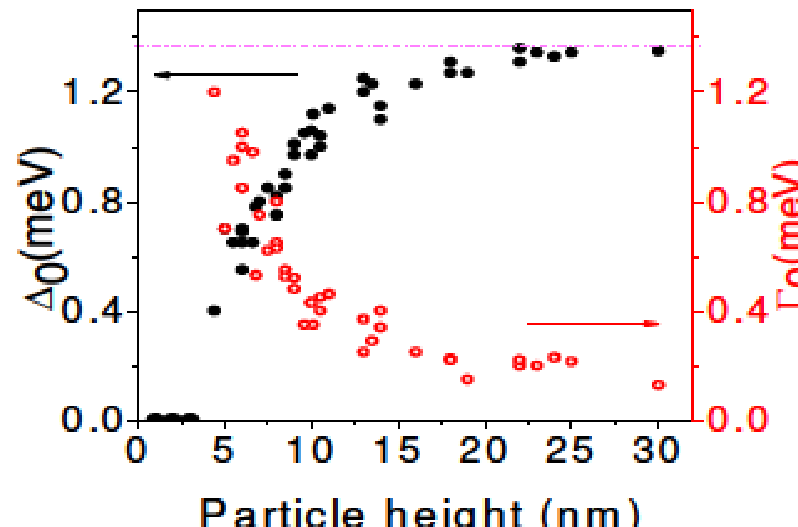
More fun?

Why not

Ribeiro,
Dresden



Pb



Beyond
mean field

Beyond mean field

Quantum Fluctuations

Random Phase Approx

Richardson Eqs

Thermal fluctuations

Path Integral
Static Path Approx

Muhlschlegel, Scalapino (1972)

Disorder, Coulomb....

Larkin, Gorkov ....

Fluctuations → $T < T_c$ finite resistivity
Stronger e-e interaction

T=0 deviations from mean field

Richardson's equations

Von Delft, Braun, Dukelsky, Marsiglio, Sierra, Smith, Ambegaokar

$$-\frac{1}{\lambda d} + \sum_{j=1}^{m}{}' \frac{1}{E_i - E_j} = \frac{1}{2}\sum_{k=1}^{n} \frac{1}{E_i - \epsilon_k} \qquad i = 1, \ldots, m$$

Ground state energy

$$E = 2\sum_{i=1}^{m} E_i + \sum_{B} \epsilon_B$$

Expansion in δ/Δ_0

$$\Delta^b = 2\Delta_0 - d\sqrt{1 + \frac{\Delta_0^2}{D^2}} + \frac{d\Delta_0}{D}[1 + \phi(\lambda)]$$

$D \equiv E_D$

$d \equiv \delta$

Richardson ~ 1968,
Yuzbashyan, Altshuler ~ 2005

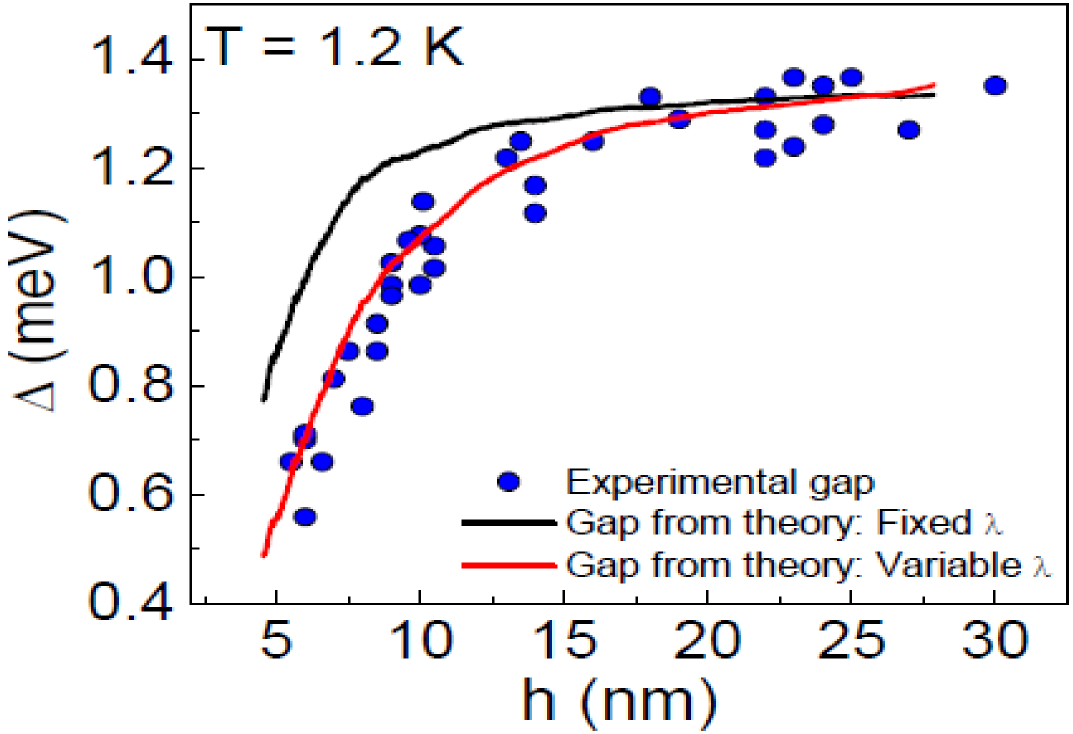
T=0

BCS size effects

Part I

Deviations BCS

Richardson



Thermal fluctuations → Path integral

$$Z=\int \mathcal{D}c^{\dagger}\mathcal{D}c\; e^{-\int_0^{\beta} d\tau\left[\sum_{k,\sigma} c^{\dagger}_{k,\sigma}(\partial_\tau+\varepsilon_k)c_{k,\sigma}-\lambda\delta\left(\sum_k c^{\dagger}_{k,1}c^{\dagger}_{-k,-1}\right)\left(\sum_{k'} c_{-k',-1}c_{k',1}\right)\right]}$$

$$=\int \mathcal{D}\Delta^{\dagger}\mathcal{D}\Delta\mathcal{D}c^{\dagger}\mathcal{D}c\; e^{-\int_0^{\beta} d\tau\left\{\sum_{k,\sigma} c^{\dagger}_{k,\sigma}[\partial_\tau+\varepsilon_k]c_{k,\sigma}+\sum_k\left(c^{\dagger}_{k,1}c^{\dagger}_{-k,-1}\Delta(\tau)+\Delta^{\dagger}(\tau)c_{-k,-1}c_{k,1}\right)+(\lambda\delta)^{-1}\Delta^{\dagger}(\tau)\Delta(\tau)\right\}}$$

Hubbard-Stratonovich transformation

0 d grains

Δ homogenous

↔

Static path approach (SPA)

Scalapino et al. 70's

$$\frac{Z}{Z_0}=\int d|\Delta|\;|\Delta|\;e^{-\beta\mathcal{A}(|\Delta|)}$$

$$\xi_k=\sqrt{\varepsilon_k^2+\Delta^{\dagger}\Delta}$$

$$\mathcal{A}\left(|\Delta|\right)=\left\{(\lambda\delta)^{-1}|\Delta|^2+\sum_{k'}\left[\left(|\varepsilon_{k'}|-\xi_{k'}\right)-\frac{2}{\beta}\log\left(\frac{e^{-\beta\xi_{k'}}+1}{e^{-\beta|\varepsilon_{k'}|}+1}\right)\right]\right.$$

$T \sim T_c$

Thermal fluctuations

Static Path Approach

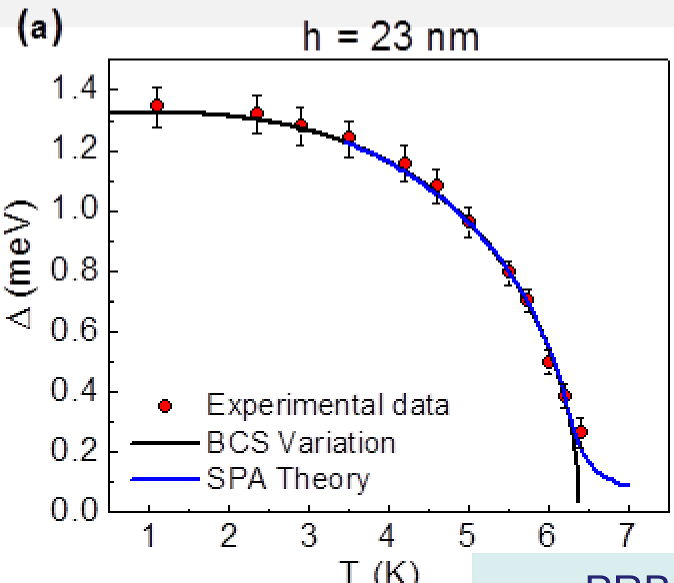
BCS finite size effects

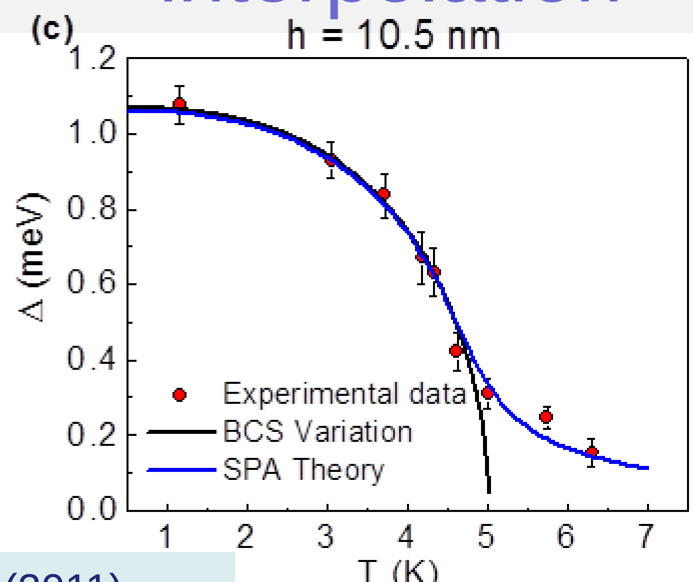
Part I

Quantum Fluctuations

Richardson formalism

$\lambda(T)$ simple interpolation





PRB 84,104525 (2011)
Editor's Suggestion

Quantum + Thermal?

Any δ/Δ_0
T=0

Richardson solution

$\delta/\Delta_0 << 1$
Any T

SPA+RPA?

Coulomb?

Dynamical phonons?

Divergences at intermediate T

Rossignoli and Canosa
Ann. of Phys. 275, 1, (1999)

BCS OK $\delta/\Delta_0 \sim 1/2$

RPA+SPA ,Ribeiro and AGG, **Phys. Rev. Lett. 108, 097004 (2012)**

Where's the problem?

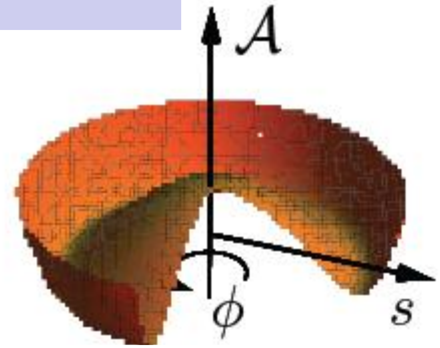
Of course the (zero modes) coordinates!!!

Castellani, et al. PRL 78, 1612 (1997)

$$\Delta(\tau) = s(\tau) e^{i\phi(\tau)}$$

$$s^2(\tau) = s_0^2 + \delta s^2(\tau)$$

$$\phi(\tau) = \phi_0 + 2\pi M \tau/\beta + \delta\phi(\tau)$$



$$\mathcal{A}[s, \phi, M] = \mathcal{A}_0(s_0) + i\pi \sum_k \left(1 - \frac{\varepsilon_k}{\xi_{0k}}\right) \frac{1}{\beta} M + \left(\sum_k \frac{s_0^2}{2\xi_{0k}^3}\right) \frac{1}{\beta^2} (\pi M)^2 + \frac{1}{2} \sum_{m \neq 0} \begin{pmatrix} \tilde{s}^2_{-m} \\ \phi_{-m} \end{pmatrix} . \Xi(s_0)_m \cdot \begin{pmatrix} \tilde{s}^2_m \\ \phi_m \end{pmatrix}$$

$$s_m^2 = \frac{1}{\beta} \int d\tau \, e^{i\Omega_m \tau} \delta s^2(\tau)$$

$$\phi_m = \frac{1}{\beta} \int d\tau \, e^{i\Omega_m \tau} \delta\phi(\tau)$$

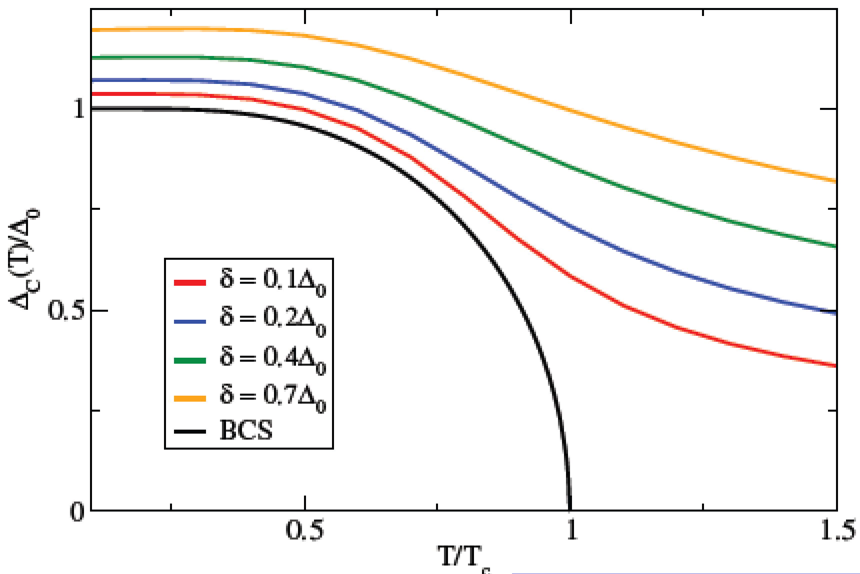
$$\tilde{s}_m^2 = \left[\beta \sum \frac{1}{2\xi_{0k}} \tanh\left(\frac{\xi_{0k}}{2}\right)\right] s_m^2$$

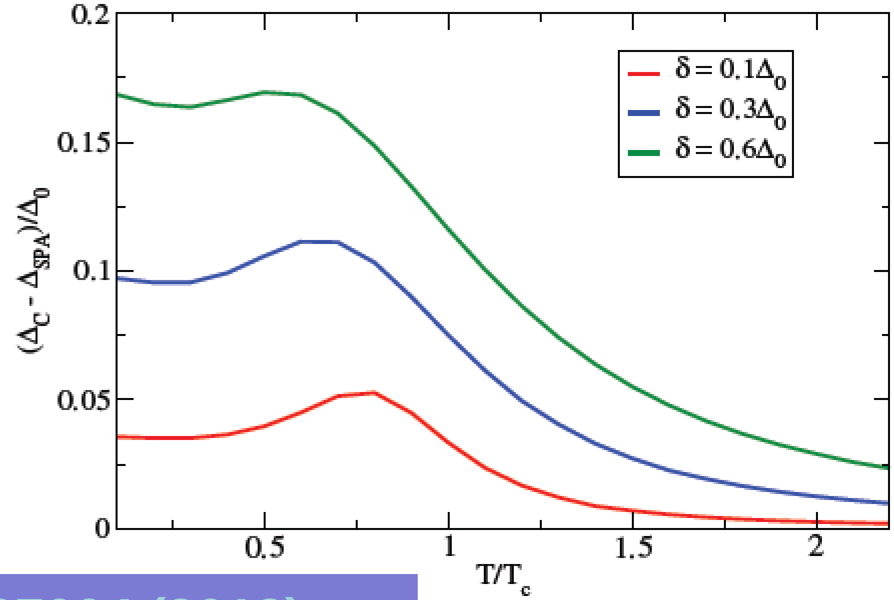
$$Z/Z_0 = \int_0^\infty ds_0^2 \; e^{-\beta[\mathcal{A}_0(s_0)+\mathcal{A}_1(s_0)]}$$

$$\mathcal{A}_1[s_0] = \frac{1}{2}\int d\nu \left[n_b(\nu) - \frac{1}{\beta\nu}\right]\frac{1}{2\pi i}\left\{\ln\left[\widetilde{C}(\nu+i0^+)\right] - \ln\left[\widetilde{C}(\nu-i0^+)\right]\right\}$$

$$\widetilde{C}(z) = (-z^2+4s_0^2)(-z^2)\left[\int_D d\varepsilon\,\varrho(\varepsilon)\frac{r(\xi)}{-z^2+(2\xi)^2}\right]^2 + (-z^2)\left[\int_D d\varepsilon\,\varrho(\varepsilon)\frac{2\varepsilon\, r(\xi)}{-z^2+(2\xi)^2}\right]^2$$

$$r(\xi) = \frac{1}{2\xi}\tanh\left(\frac{\beta\xi}{2}\right)$$

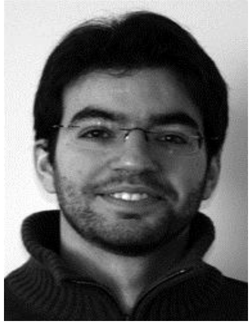




PRL 108, 097004 (2012)

Charging effects?

The same



Perturbative

$$\Xi_m^{\phi\phi} = \sum_k r_k \frac{2\beta s_0^2 \Omega_m^2}{\Omega_m^2 + (2\xi_{0k})^2} \sim \frac{\beta}{\delta}\Omega_m^2$$

Charging effects

$$\longrightarrow \quad \delta^{-1}\int_0^\beta d\tau\,(\partial_\tau \delta\phi)^2$$

Non perturbative

$$\phi(\tau) = \phi_0 + 2\pi M\tau/\beta + \delta\phi(\tau)$$

Odd-Even at T=0

Charging

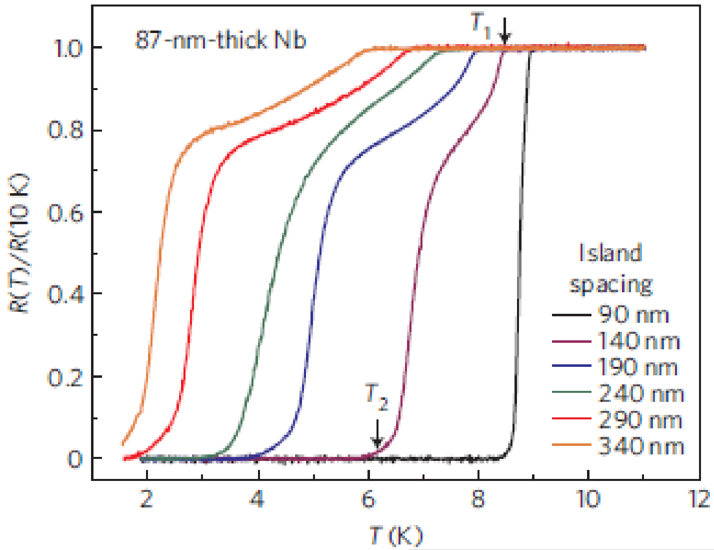
=

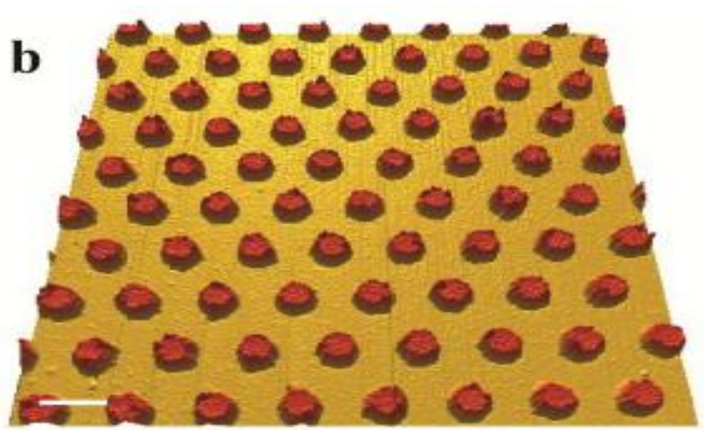
fluctuations

NEXT

Enhancement SC?

Josephson junctions

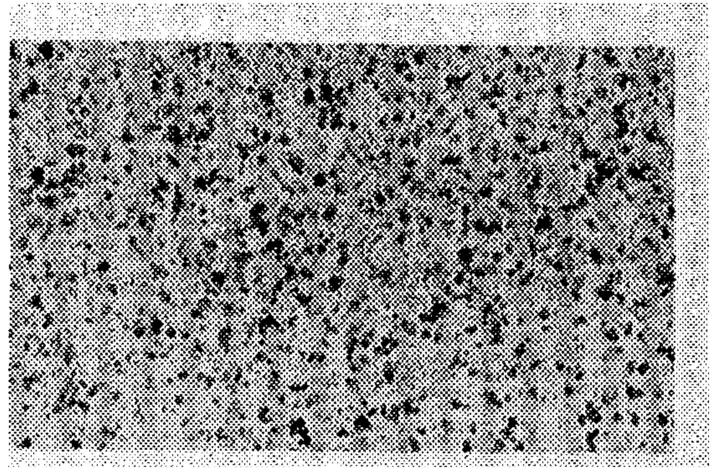




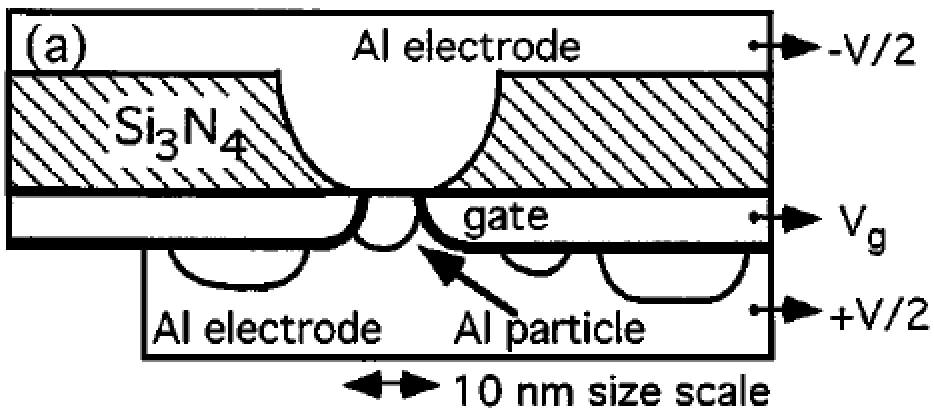
Mason, Goldbart et al, Nature Physics 8 59 (2012)

+Experimental control

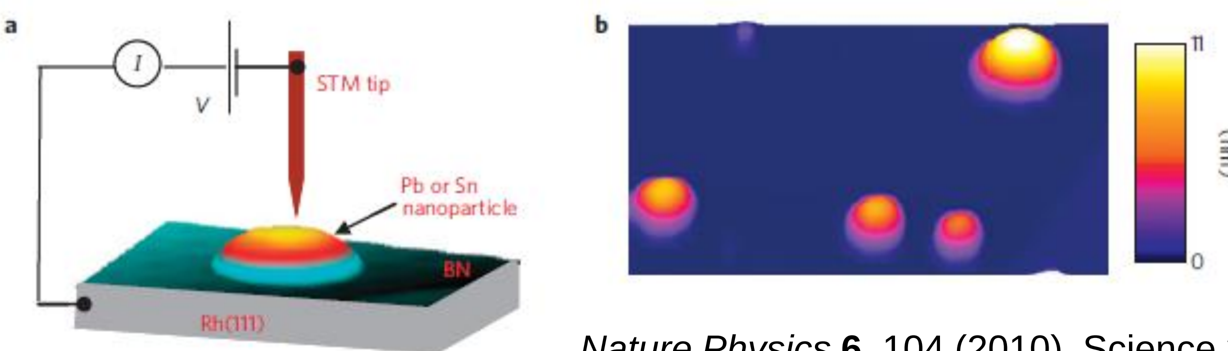
+Predictive power



1966



1995



Now

Nature Physics **6**, 104 (2010), Science 324, 1314 (2009).

Finite Size
+
Strong interactions ?

Tough for even
conventional superconductors

Holographic
superconductivity in
confined geometries?

Benson
Way

Jorge
Santos

Holographic principle

Maldacena’s conjecture

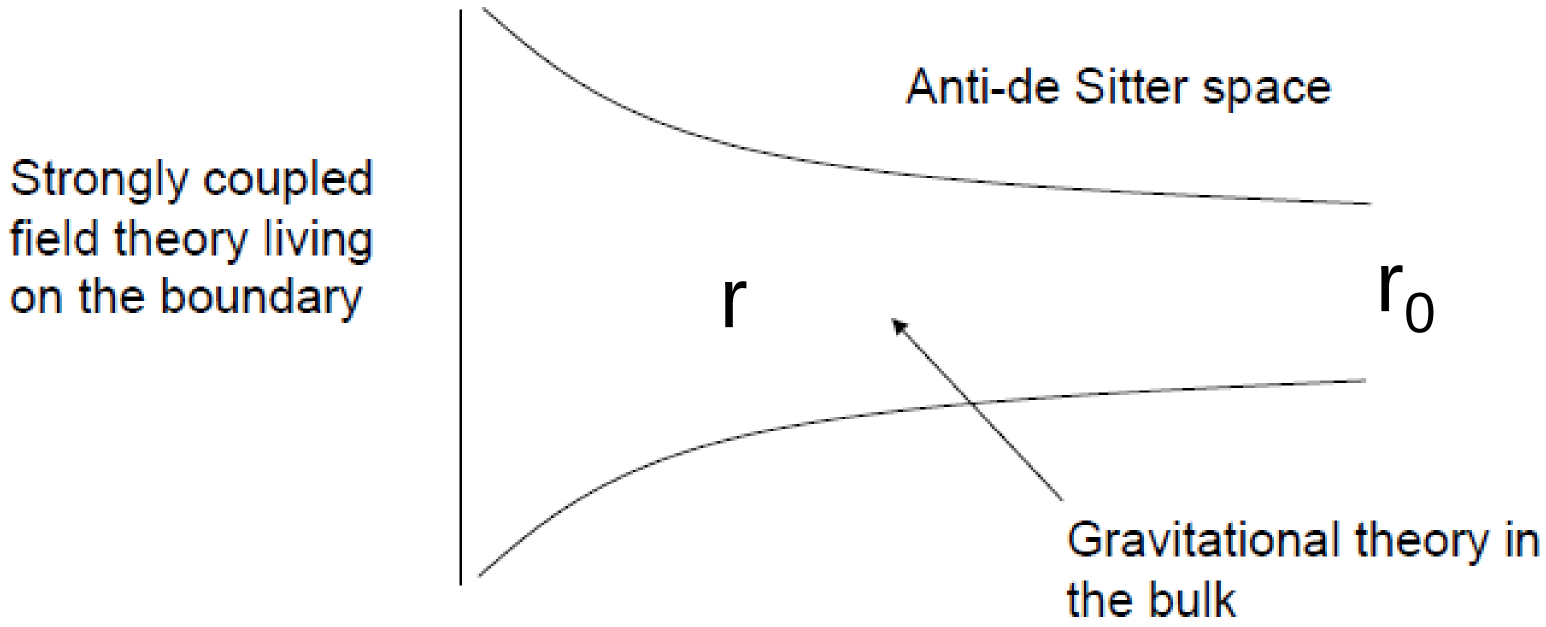
AdS/CFT correspondence

t’Hooft, Susskind, Weinberg, Witten....

Strongly coupled field theory in d		Weakly coupled gravity in d+1
N=4 Super-Yang Mills CFT	⟺	Anti de Sitter space AdS

Extra dimension?

Geometrization of Wilson RG



Holography beyond string theory

2003	QCD Quark gluon plasma Gubser, Son
2008	Holographic superconductivity Hartnoll, Herzog, Horowitz
2012	Quantum criticality, non-equilibrium.. Zaanen, Sachdev, Philips

Easy to compute in the gravity dual & Detailed dictionary

An answer looking for a question

H = ?

I do not know

Complex scalar

I know that

Spontaneous breaking U(1) at low T

Finite μ

Simplest dual gravity theory

$$S = \int d^4x \sqrt{-g}\left[R + \frac{6}{L^2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\psi|^2 - m^2|\psi|^2\right]$$

$$D = \nabla - iqA$$

ψ ≡ complex scalar

Metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

$$= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dR^2 + R^2 d\theta^2)$$

$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_0^3}{r^3}\right),$$

Equations of motion:

$$\partial_r^2|\psi| + \frac{1}{r^2 f}\partial_x^2|\psi| + \left(\frac{f'}{f} + \frac{2}{r}\right)\partial_r|\psi| + \frac{1}{f}\left(\frac{A_t^2}{f} - m^2\right)|\psi| = 0$$

$$\partial_r^2 A_t + \frac{1}{r^2 f}\partial_x^2 A_t + \frac{2}{r}\partial_r A_t - \frac{2|\psi|^2}{f}A_t = 0$$

Boundary conditions:

$r = r_0$ **$r \to \infty$**

$A_t = 0$

$$|\psi| = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2} + O\left(\frac{1}{r^3}\right)$$

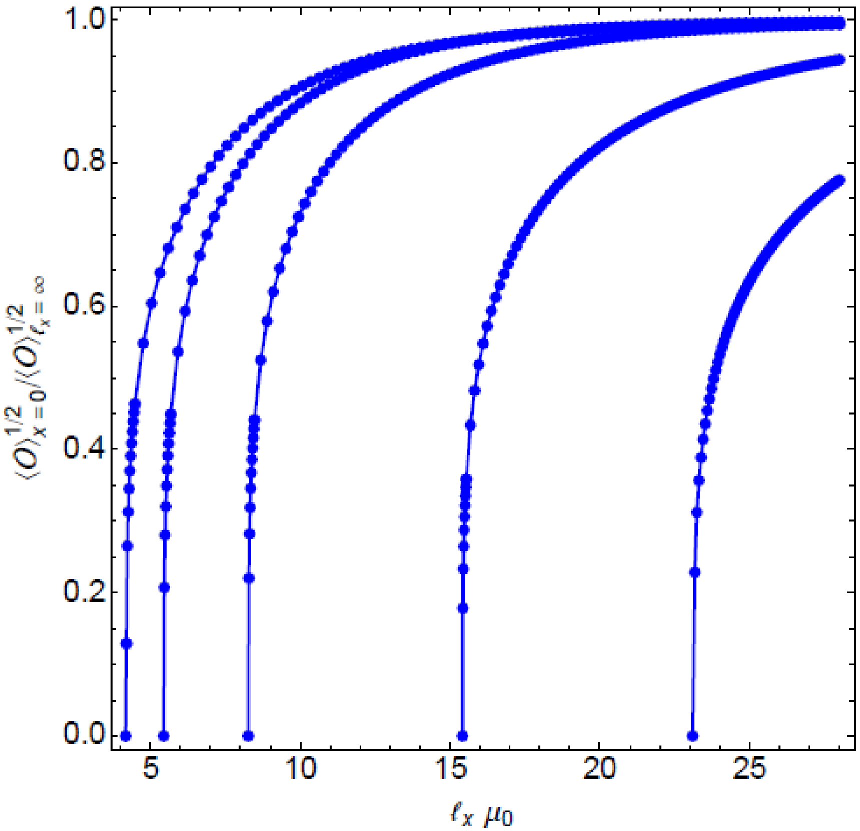
$$A_t = \mu - \frac{\rho}{r} + O\left(\frac{1}{r^2}\right)$$

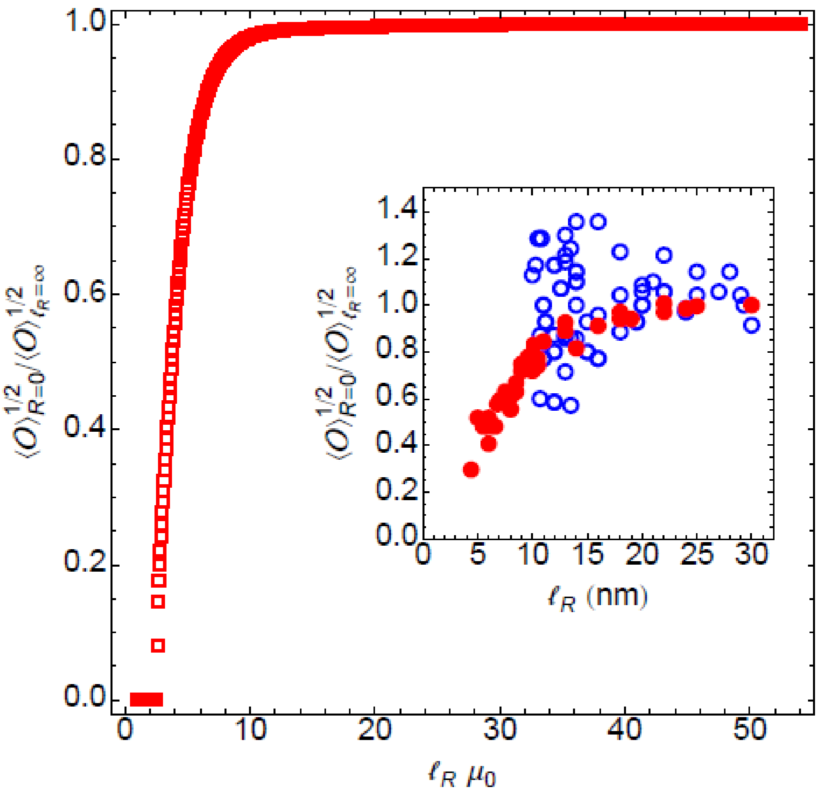
How small?

$$\mu(x) = \mu_0\left[\frac{1 - \epsilon + \epsilon\cosh\left(\frac{2x}{\sigma}\right) + \cosh\left(\frac{\ell_x}{\sigma}\right)}{\cosh\left(\frac{2x}{\sigma}\right) + \cosh\left(\frac{\ell_x}{\sigma}\right)}\right]$$

Dictionary:

$$\langle\mathcal{O}\rangle = \sqrt{2}\psi^{(2)}$$

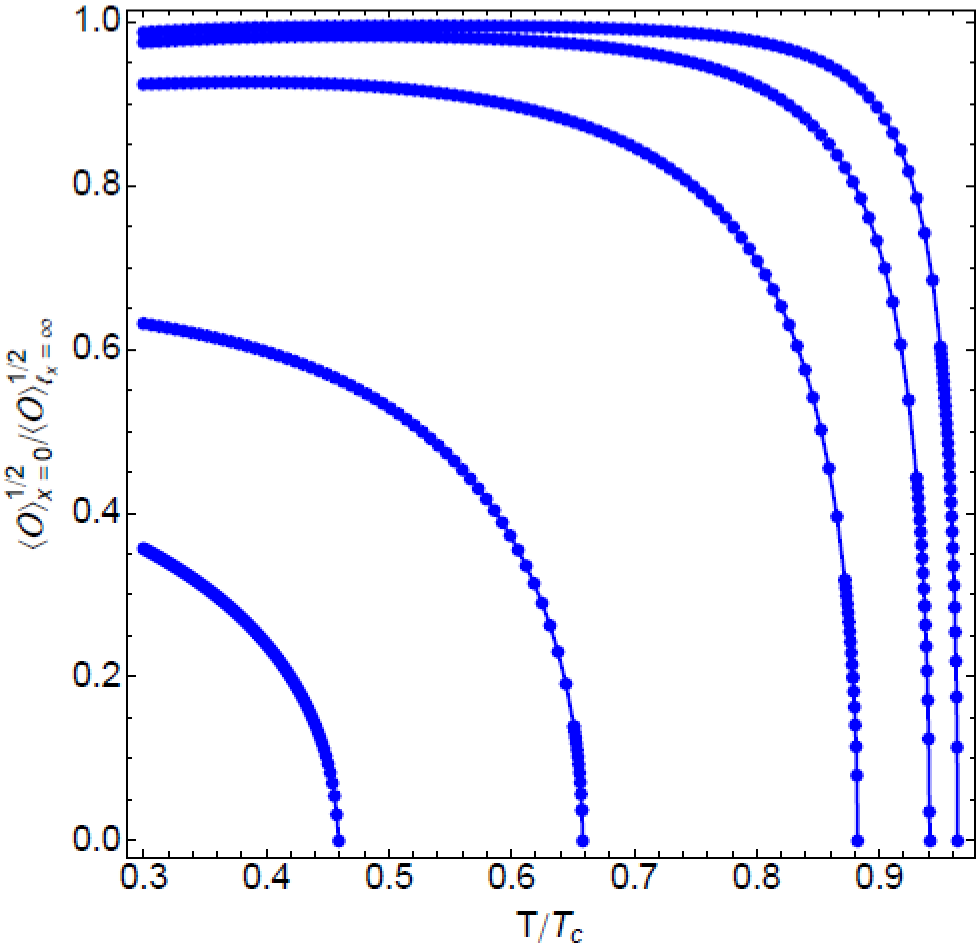




“Superconductivity” only for $l < l_c$

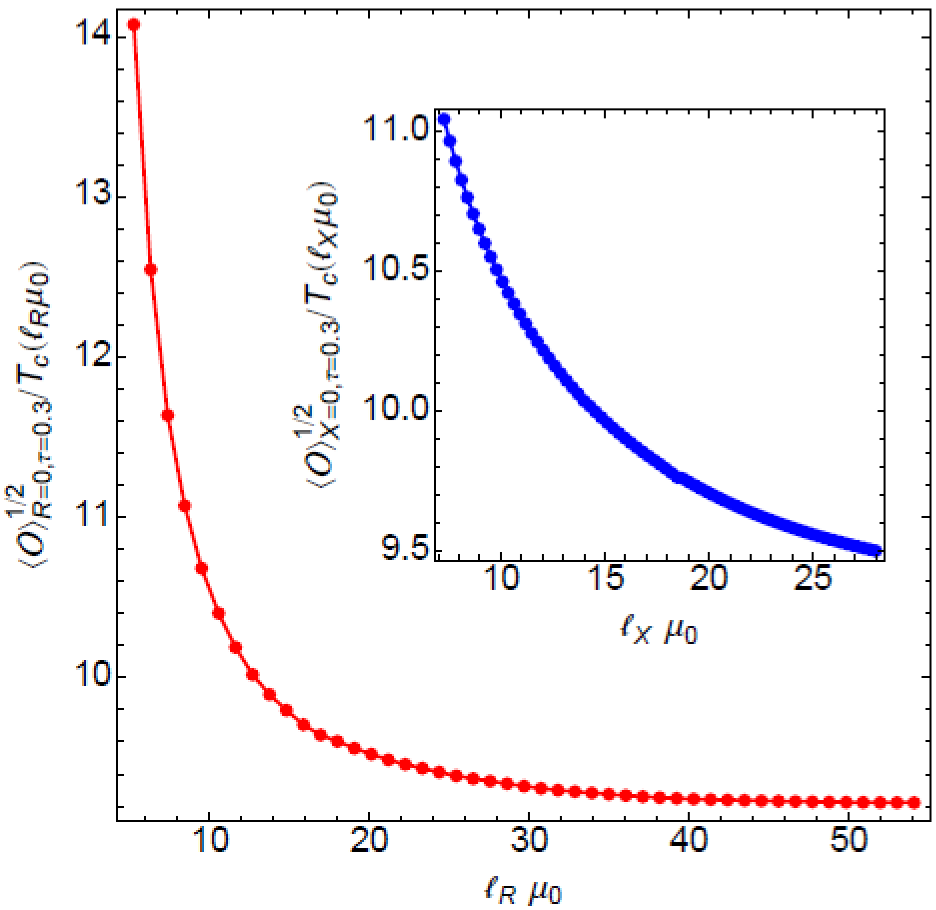
Mean field behavior

Fluctuations?



No thermal fluctuations

Large N artefact



Interactions depends on system size!

PRB, 86, 064526 (2012)

Next

Theory

Heterostructures
Collections of grains

Topology

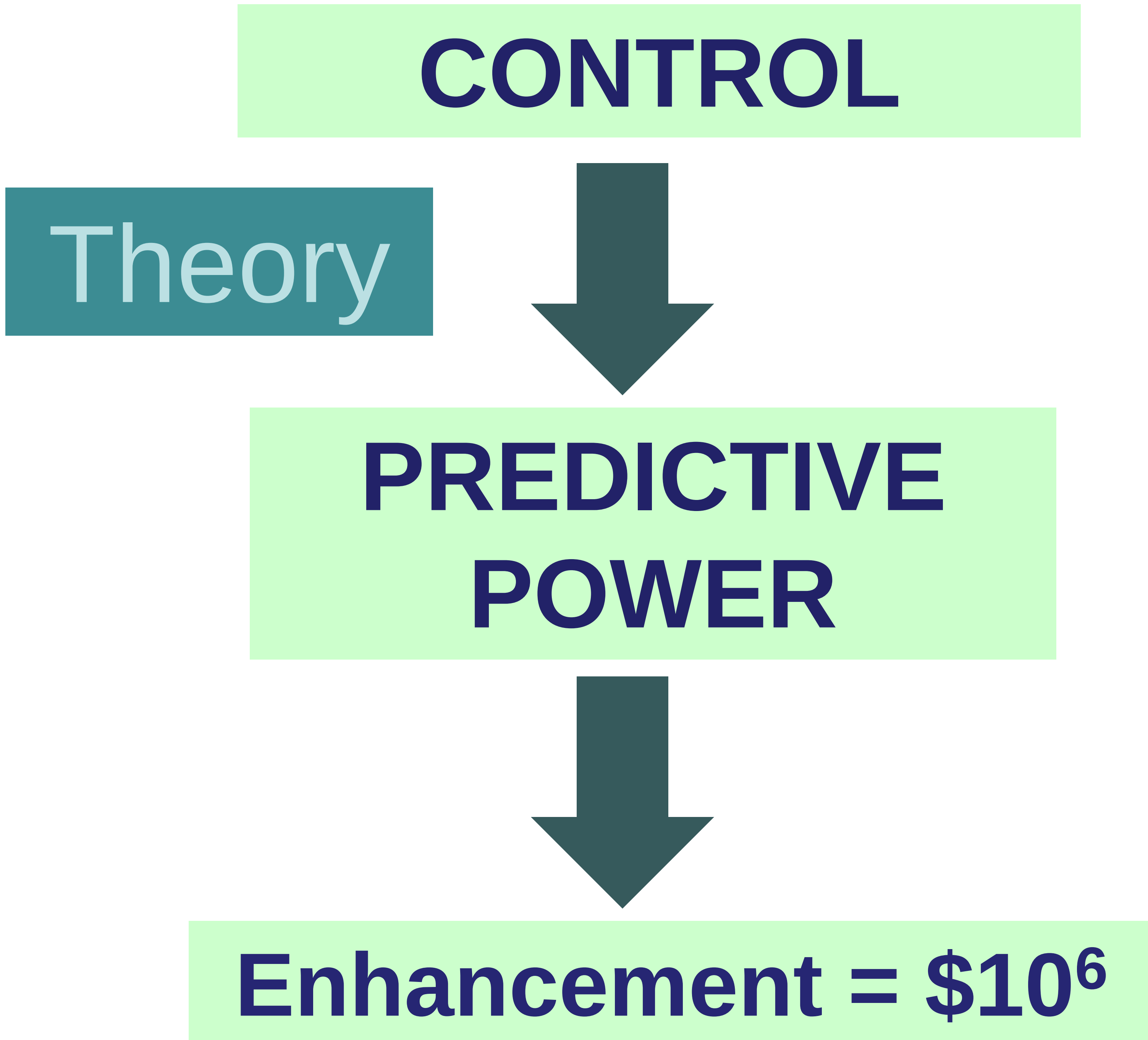
Non-equilibrium

Experiments

Control on high T_c heterostructures

Control on grains arrangements

Substantial enhancement of T_c


CONTROL
Theory
PREDICTIVE POWER
Enhancement = 10^6

THANKS!

感谢您的关注