Smaller is different and more

Antonio M. García-García
EPSRC Career Acceleration
Cavendish Laboratory, Cambridge University
http://www.tcm.phy.cam.ac.uk/~amg73/

Pedro Ribeiro
Dresden

PRB, 86, 064526 (2012)
PRL 108, 097004 (2012)
PRB 84,104525 (2011)
Editor's Suggestion
Arxiv:1212.6779

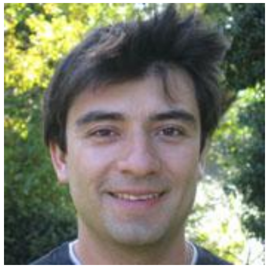
Tezuka
Kyoto

Santos & Way
Santa Barbara

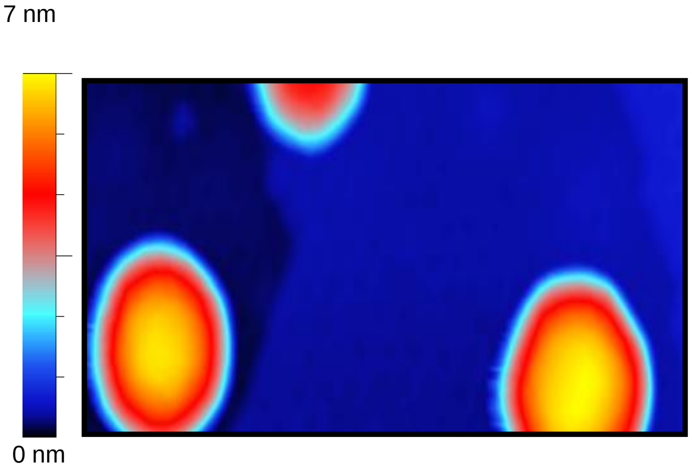
Sangita Bose
Bombay

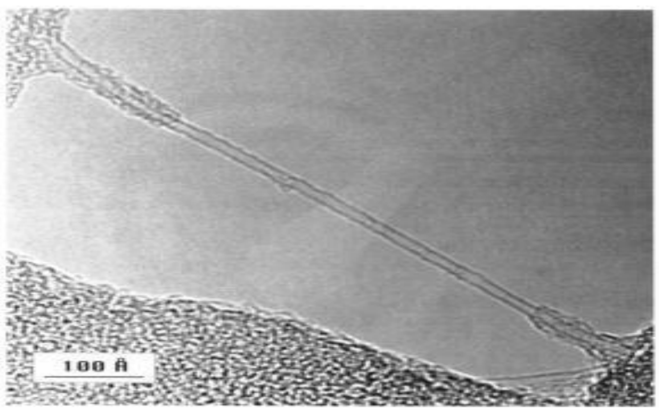
Altshuler
Columbia

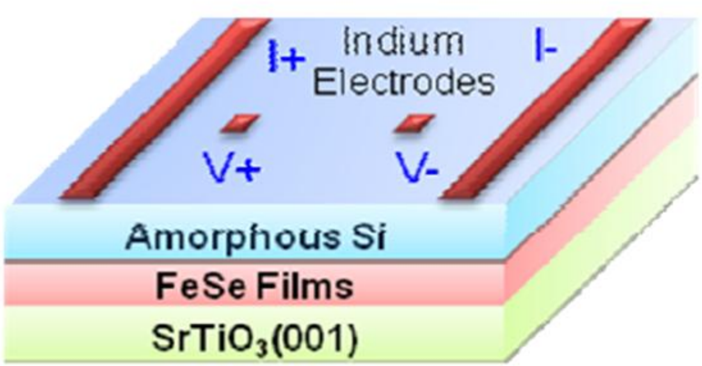
Klaus Kern
Stuttgart



Lobos
Maryland







Why?

Boring?

Mermin-Wegner theorem

Low dimensions
=
No superconducivity

Exciting?

Beauty

Meso+Super
Quantum Control
Enhancement

Enhancement?

How to enhance SC substantially?

with **control**

$\$10^6$ Question

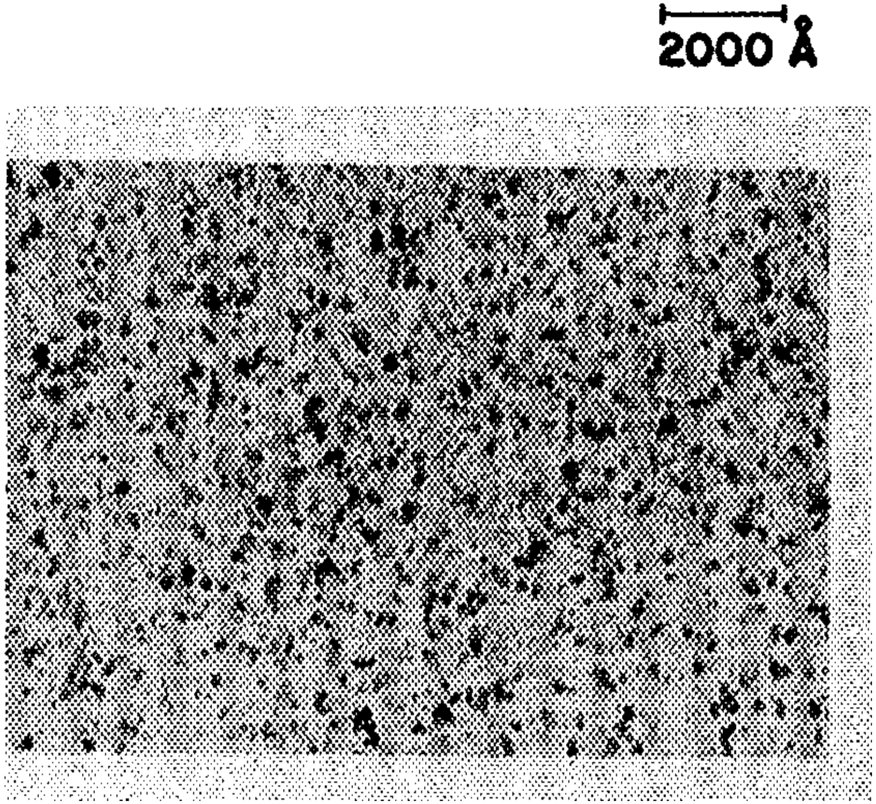
?

Mechanism of SC in cuprates?

$10 Question

Thin Films? JJ array?

Metal	T_c (°K)	T_c/T_{c0}	d (Å)	ρ_0
Al	3.0	2.6	40	0.19
Ga	7.2	6.5	···	0.20
Sn	4.1	1.1	110	0.31
In	3.7	1.1	110	0.36
Pb	7.2	1.0	···	0.53

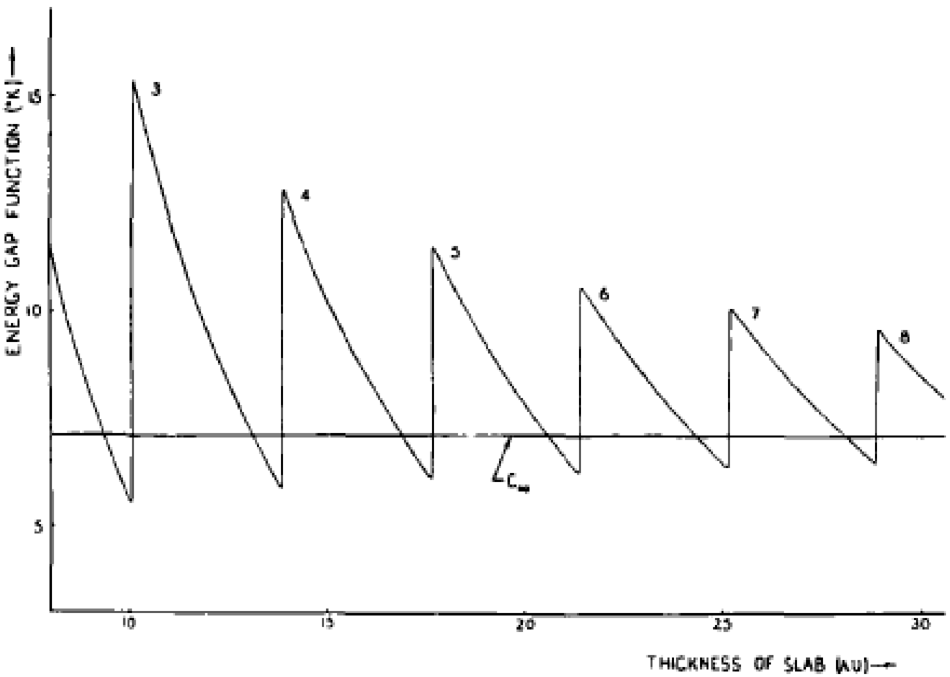


Abeles, Cohen, Cullen, Phys. Rev. Lett., 17, 632 (1966)

Crow, Parks, Douglass, Jensen, Giaver, Zeller....

A.M. Goldman, Dynes, Tinkham…

Thin Films	Single grains



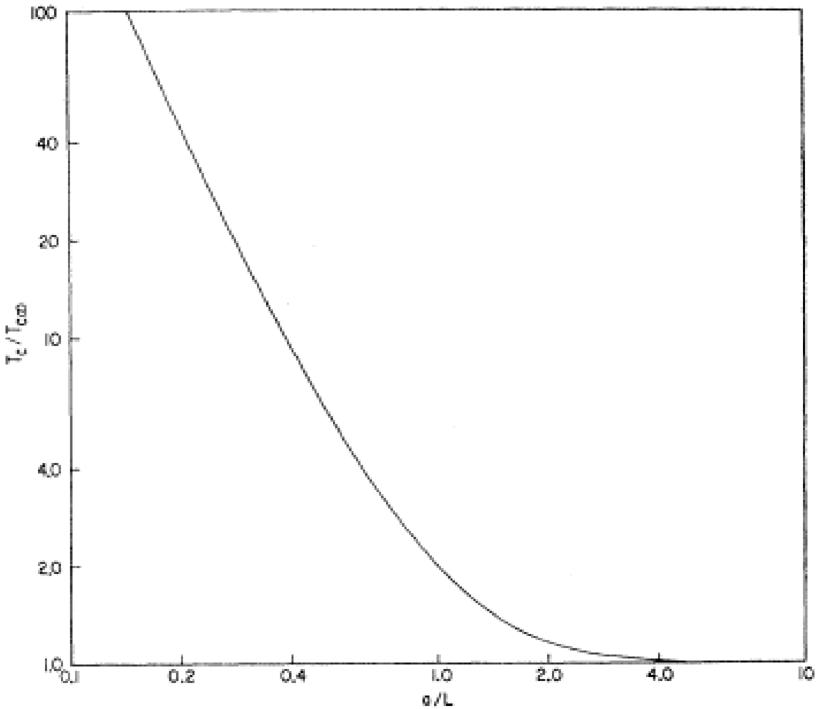


Fig. 1. $(T_c/T_{c\infty})$ versus (a/L) (see Ref. 17).

Shape Resonances

Blatt, Thompson
Phys. Lett. 5, 6 (1963)

Shell Effects

Parmenter, Phys. Rev. 166, 392 (1967)

Thinner

Smoother

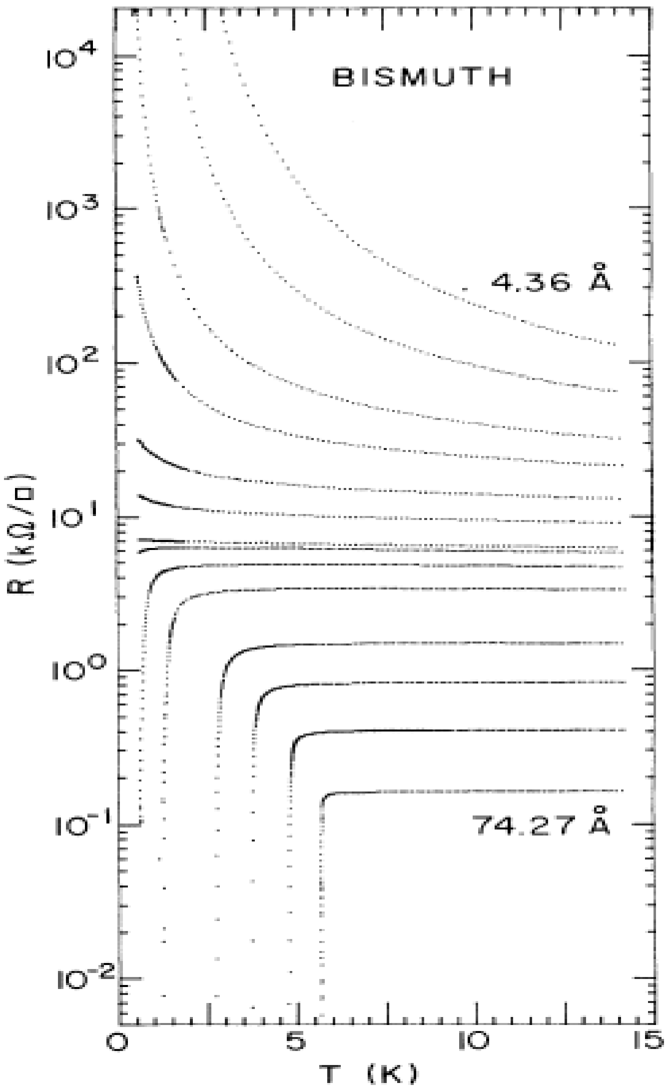
Disordered

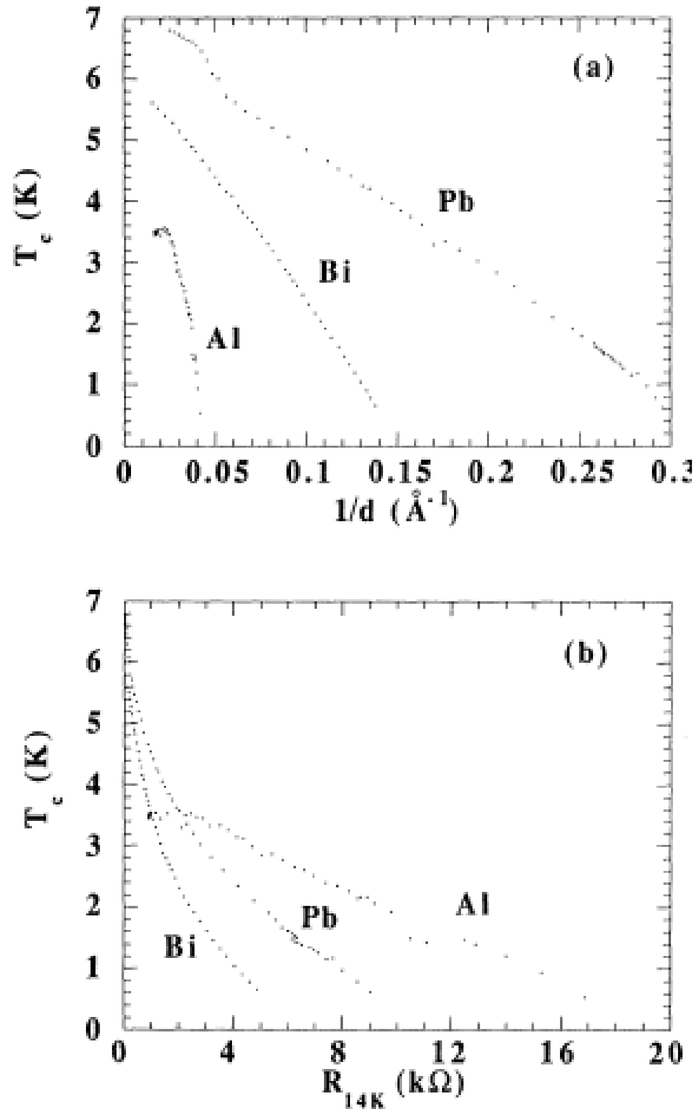
BKT

Transition

$R_N > R_q$

Vortices unbinding

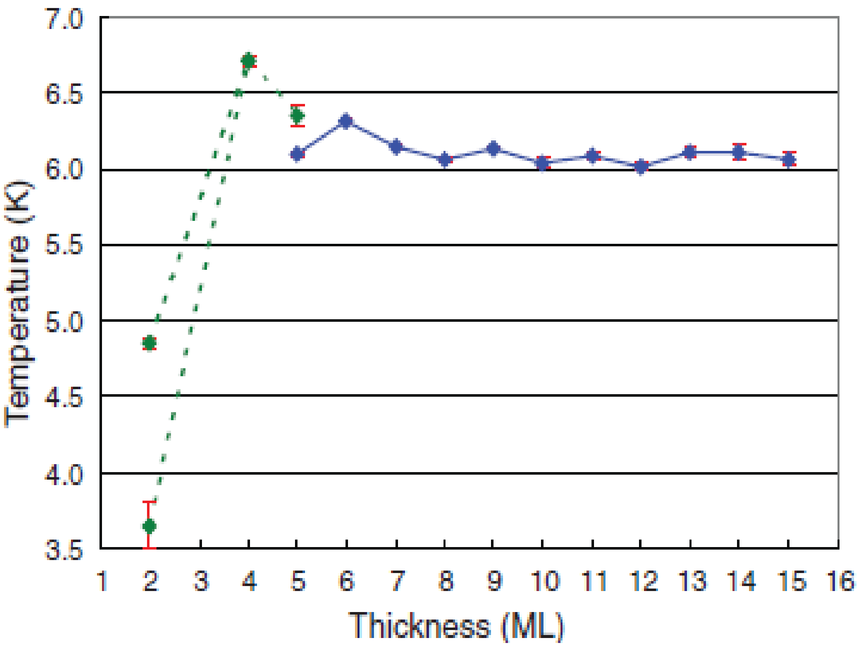




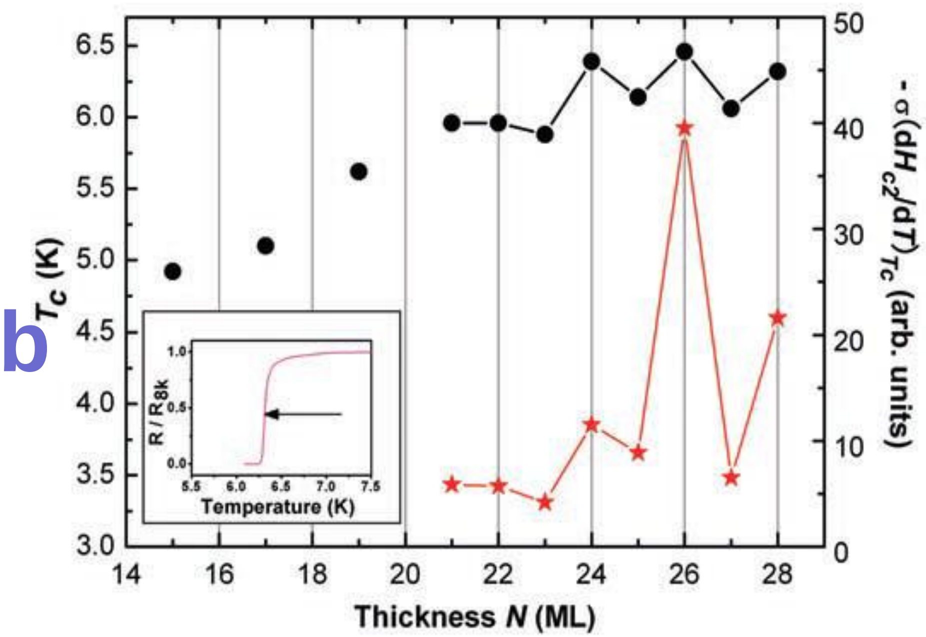
A.M. Goldman et al.

PRL 62 2180 (1989)
PRB 47 5931 (1993)

Recent Atomic scale **control**



Pb

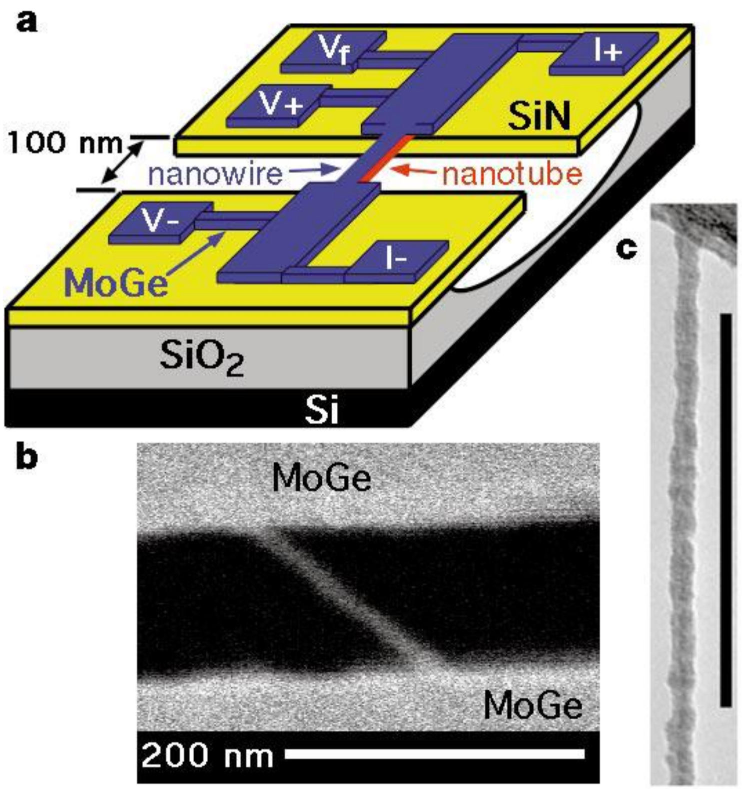


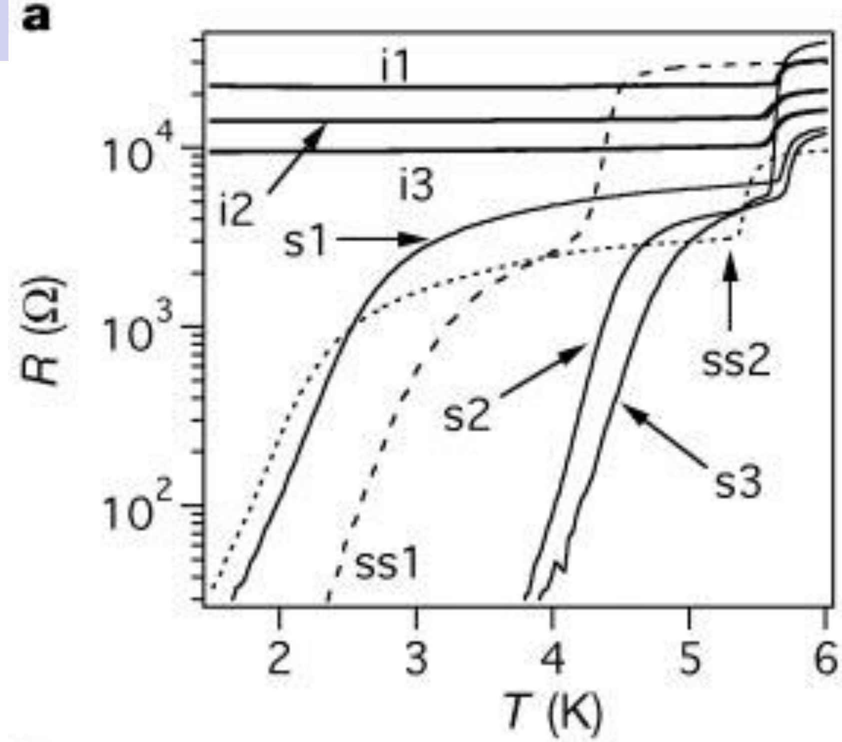
Shih et al., Science 324, 1314 (2009)

Xue et al., Science 306, 1915 (2004)

Xue et al., Nat Phys, 6 (2010),104.

Nanowires R <<ξ





Tinkham et al.
Nature 404, 971 (1990)

Superconductor Insulator transition

$$|\Delta(\mathrm{r},\mathrm{t})|e^{i\theta(r,t)}$$

Fluctuation

$$\Delta(r_0, t_0) \approx 0$$

Phase-slips

$$\theta \approx 0 \rightarrow 2\pi$$

Finite Resistence

$$R \propto e^{-S_{inst}}$$

Thermal

Langer & Ambegaokar, PR. 164, 498 (1967).

McCumber & Halperin PRB 1, 1054 (1970).

Quantum

Zaikin, A. D., Golubev, et al, PRL 78, 1552 (1997).

Instantons

Small Fluctuations

Coulomb-Gas

BKT transition

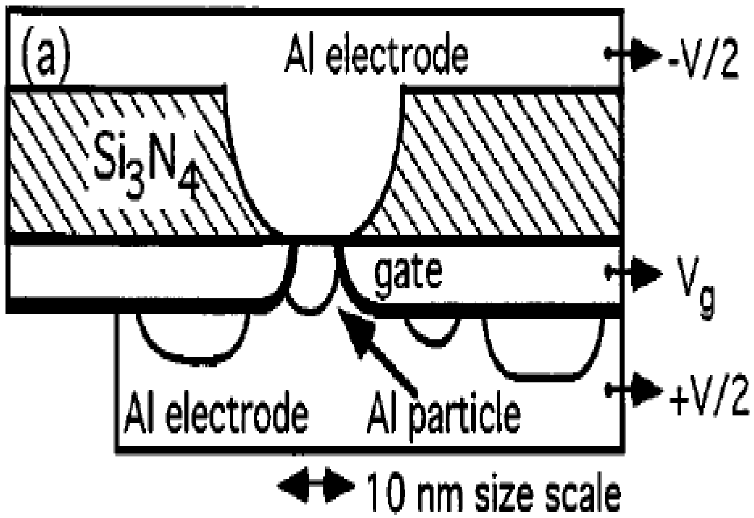
Quasi LRO

Ultrasmall Grains

T = 0

$\delta / \Delta_0 \sim 1$

von Delft, Braun, Larkin, Sierra, Dukelsky, Yuzbashyan, Matveev, Smith, Ambegaokar



Tinkham et al., PRL 74, 3244 (1995)

Superconductivity?

1959

Richardson

It's exact. I did it 20 years ago

BCS fine until $\delta / \Delta_0 \sim 1/2$

BCS sharp transition

Richardson no transition

J. von Delft et al., Phys. Rep., 345, 61 (2001)

Richardson's equations

Von Delft, Yuzbashyan, Dukelsky, Marsiglio,Braun Sierra, Ambegaokar

$$-\frac{1}{\lambda d}+\sum_{j=1}^{m}{}'\frac{1}{E_i-E_j}=\frac{1}{2}\sum_{k=1}^{n}\frac{1}{E_i-\epsilon_k}\qquad i=1,\ldots,m$$

Ground state energy

$$E=2\sum_{i=1}^{m}E_i+\sum_{B}\epsilon_B$$

Expansion in δ/Δ_0

$$\Delta^b=2\Delta_0-d\sqrt{1+\frac{\Delta_0^2}{D^2}}+\frac{d\Delta_0}{D}\left[1+\phi(\lambda)\right]$$

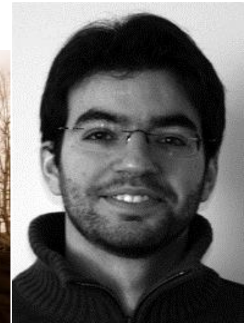
Richardson, 1968, Yuzbashyan, Altshuler, 2005

$D \equiv E_D$

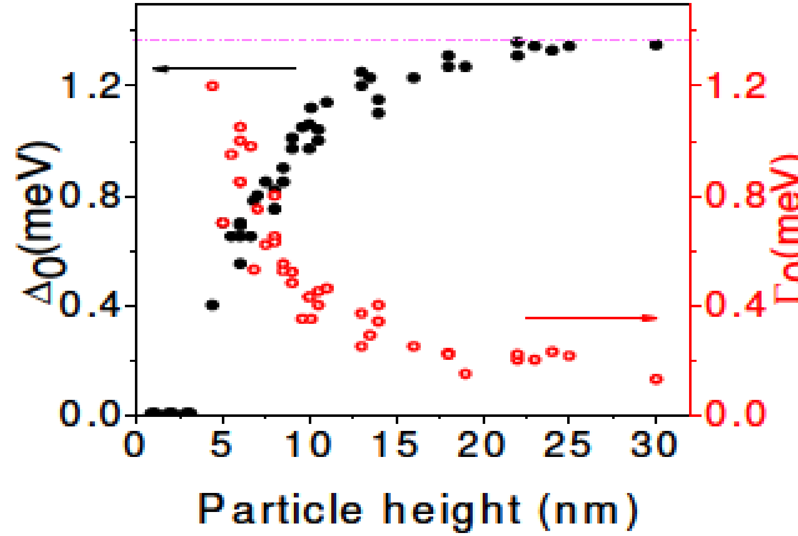
$d \equiv \delta$

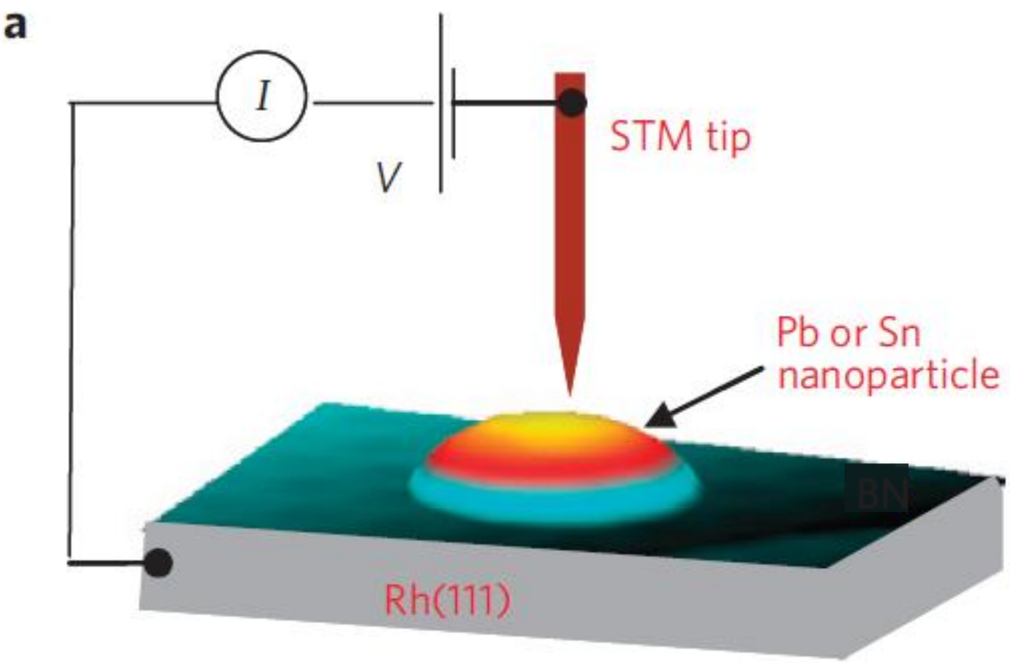
No enhancement

Kern
Stuttgart

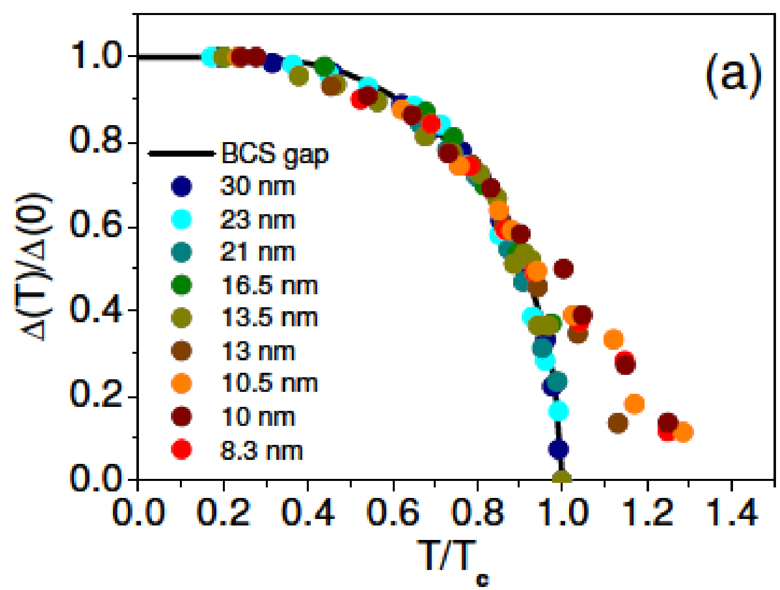


Ribeiro
Dresden



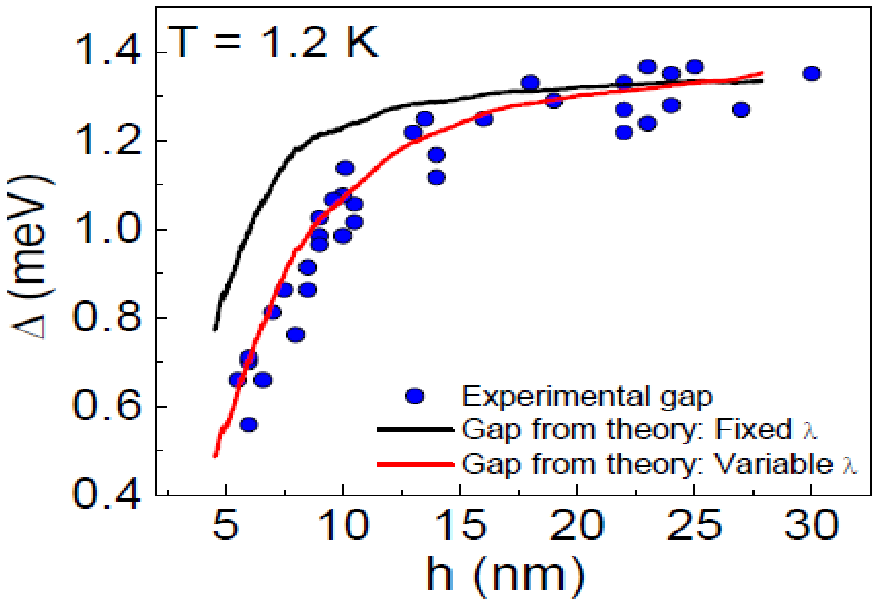


Pb

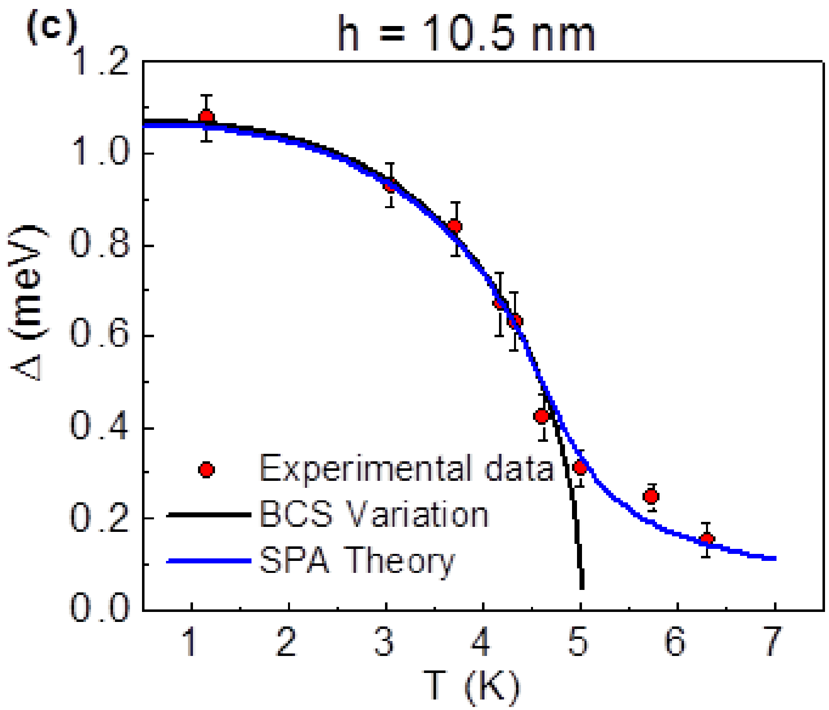


Quantum Fluctuations

Richardson's equations



and



Thermal Fluctuations

Static Path Approach

Scalapino, et al.

PRB 84,104525 (2011)
Editor's Suggestion

Quantum + Thermal? $T, \delta/\Delta_0 << 1$

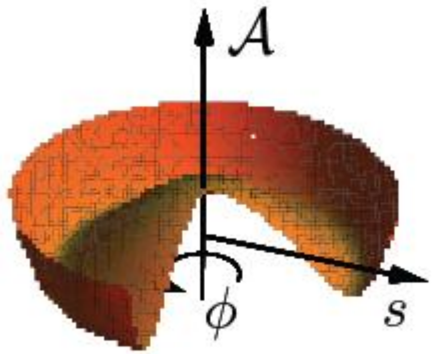
Divergences at intermediate T

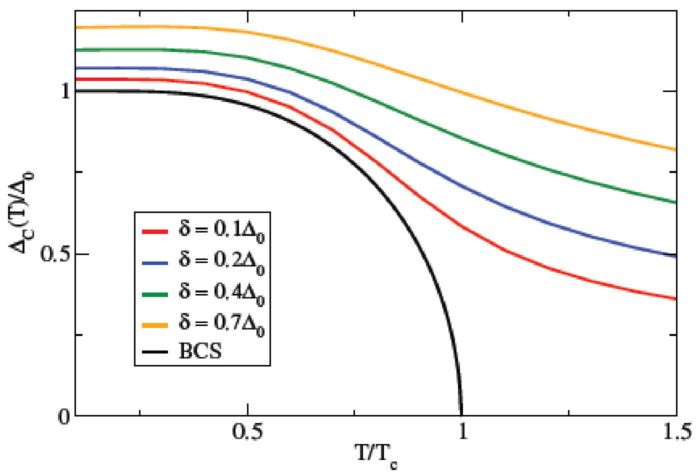
Rossignoli and Canosa
Ann. of Phys. 275, 1, (1999)

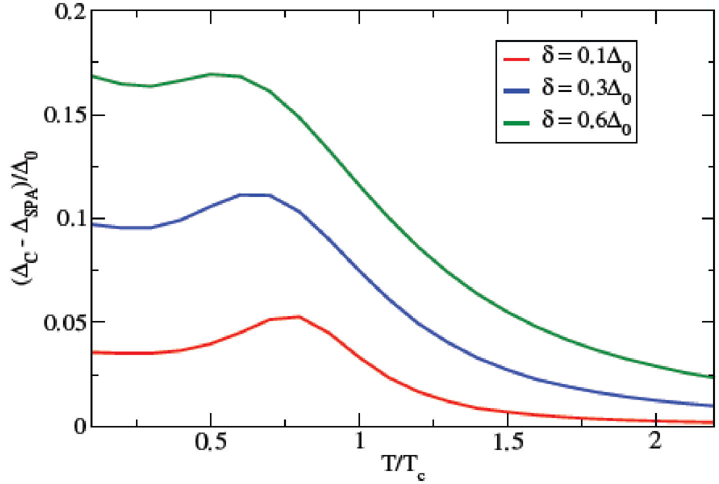
$$\Delta(\tau) = s(\tau)e^{i\phi(\tau)}$$

$$s^2(\tau) = s_0^2 + \delta s^2(\tau)$$

$$\phi(\tau) = \phi_0 + 2\pi M\tau/\beta + \delta\phi(\tau)$$



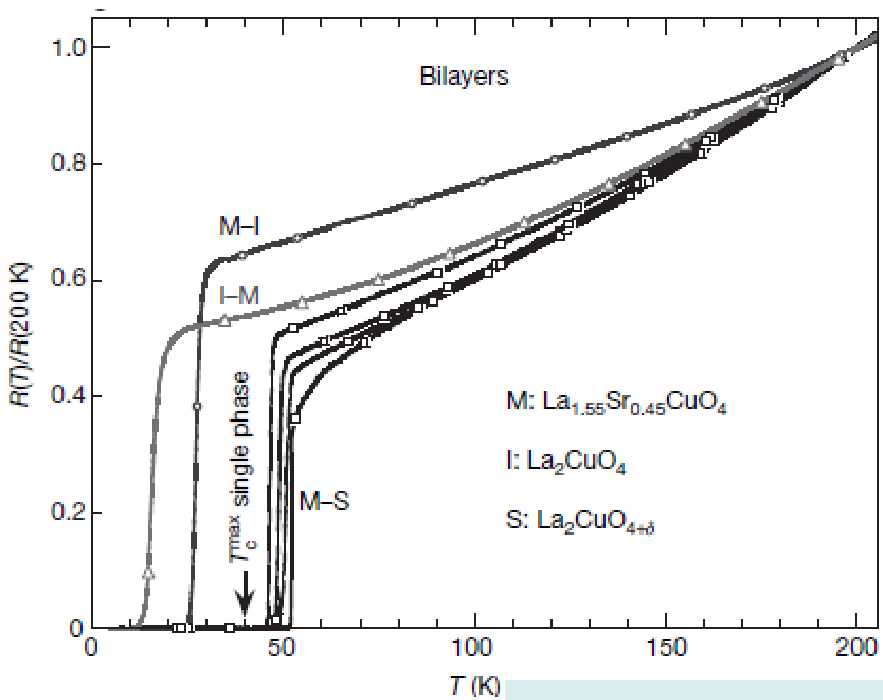


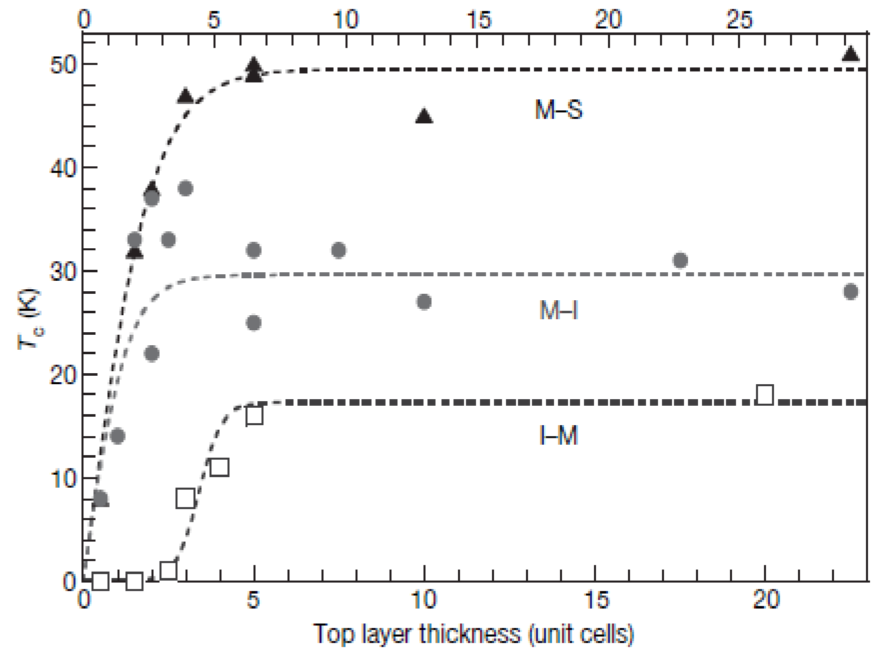


Ribeiro and AGG, **Phys. Rev. Lett. 108, 097004 (2012)**

Is enhancement of superconductivity possible?

Cuprates high T_c Heterostructures



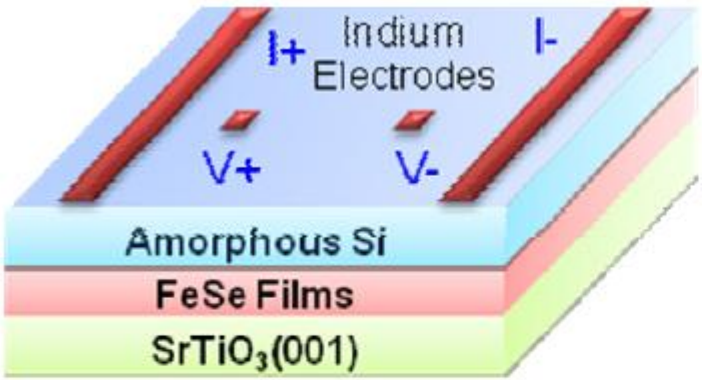


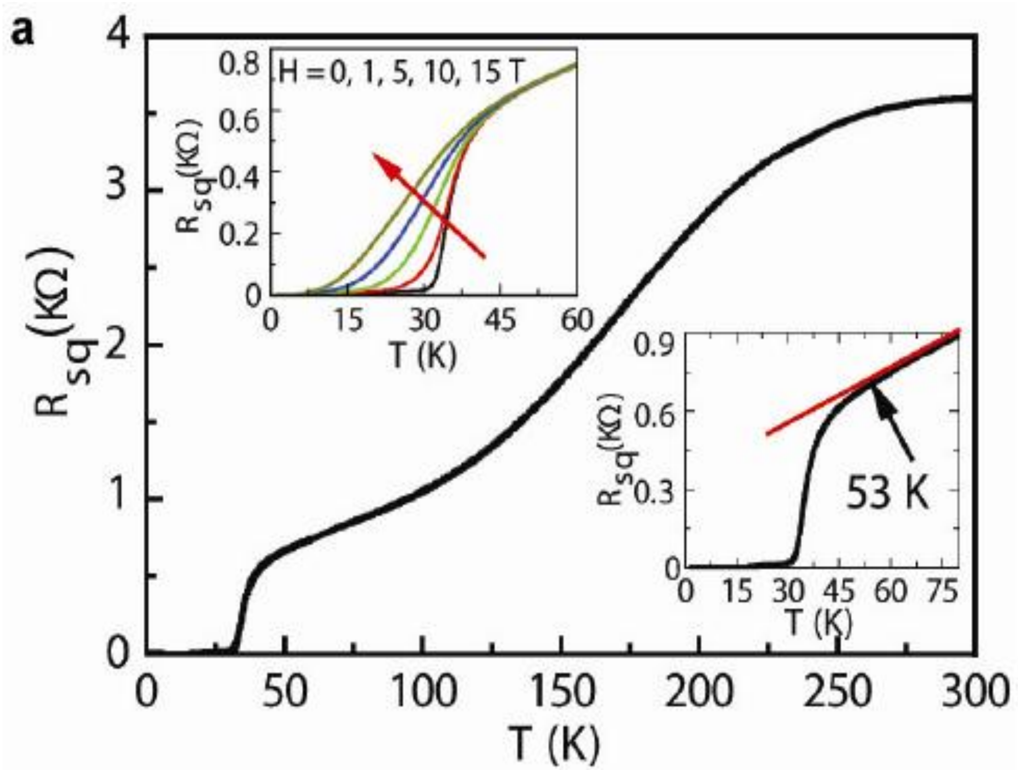
Bozovic et al., Nature 455, 782 (2008)

Higher T_c!!

Intrinsic inhomogeneities

Iron Pnictides Heterostructures

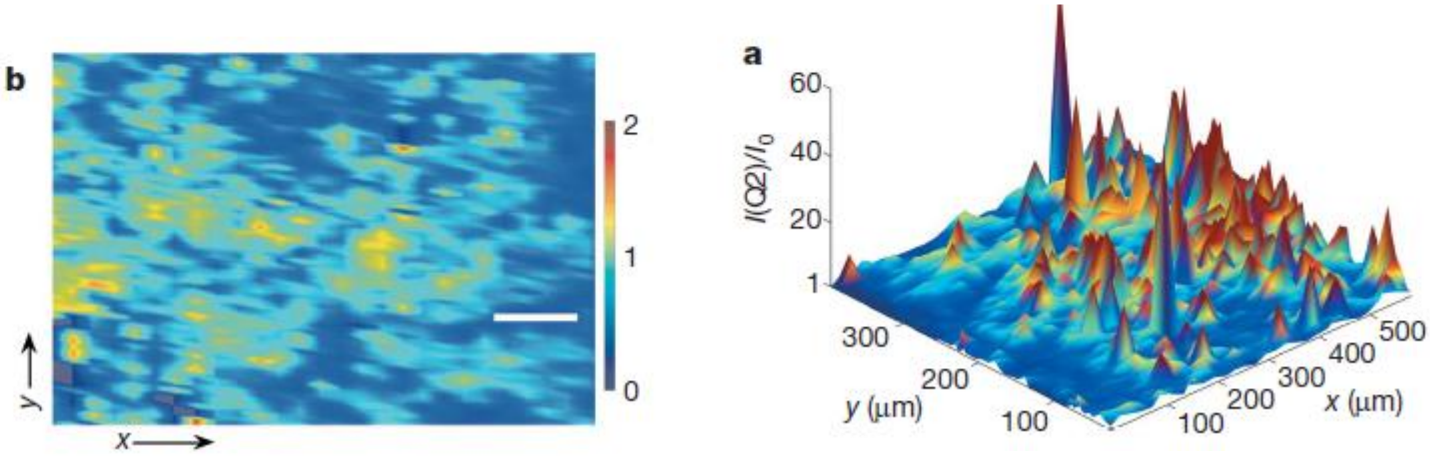




Xue et al.
Nature Communications 3, 931 (2012)

Enhancement of T_c by disorder

Fractal distributions of dopants enhances SC in cuprates



Bianconi, et al., Nature 466, 841 (2010)

Inhomogeneities ⟷ **Higher T_c**

PRL 108, 017002 (2012)

Δ >> δ Grains

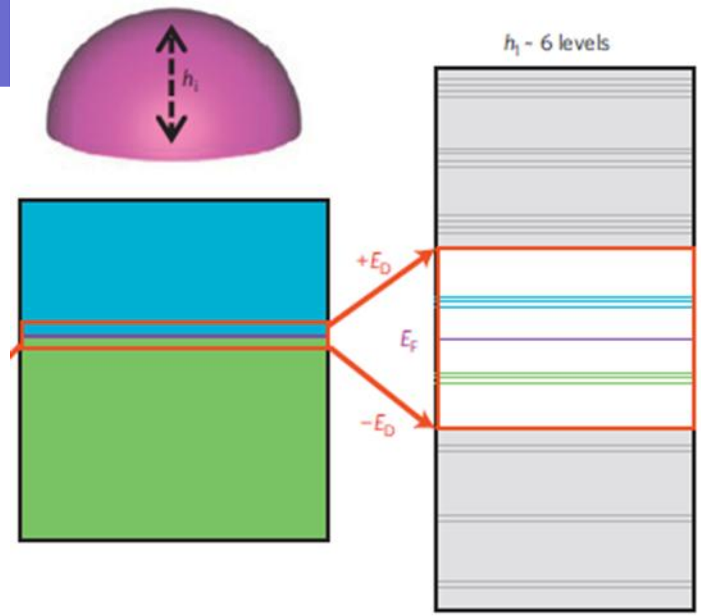
Heiselberg (2002): harmonic potentials, cold atom

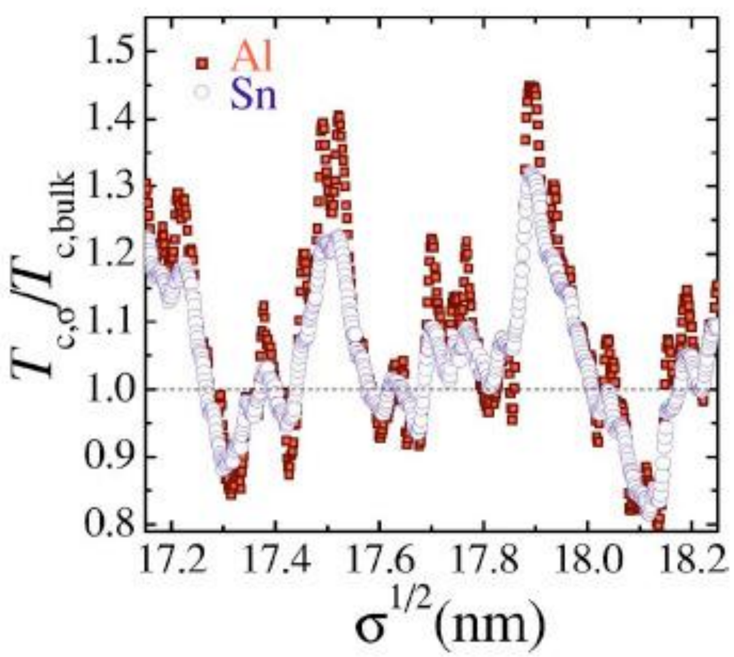
Kresin, Ovchinnikov, Boyaci (2007) : Spherical, too high T_c

Peeters, et al, (2005-): BCS, BdG in a wire, cylinder..

Devreese (2006): Richardson equations in a box

Olofsson (2008): Estimation of fluctuations in BCS





Chaotic

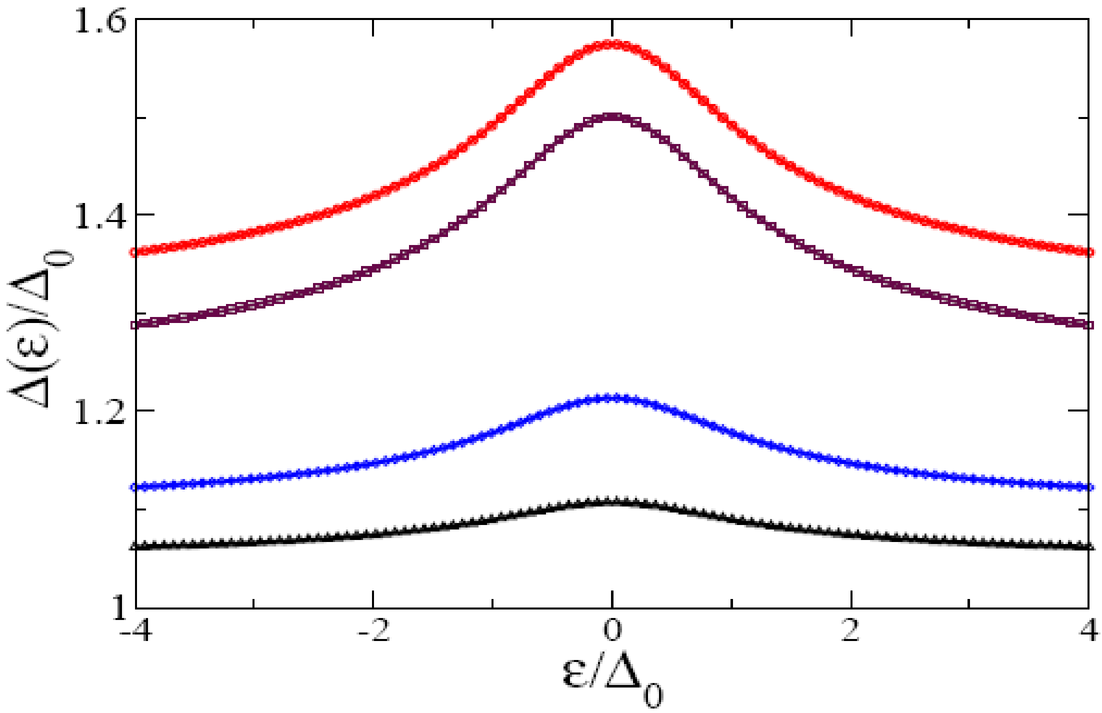
Al grain

k_F = 17.5 nm^{-1}

Δ_0 = 0.24mV

For L< 9nm leading correction comes from density-density

AGG et al., PRL 100, 187001 (2008)
AGG et al., PRB 83, 014510 (2011)



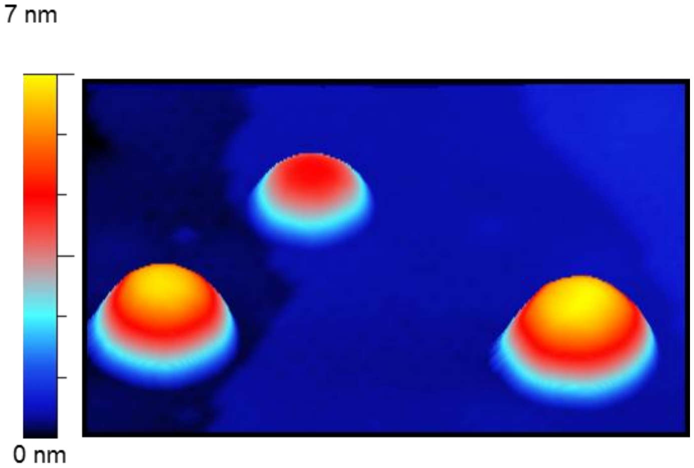
L = 6nm, Dirichlet, δ/Δ_0=0.67

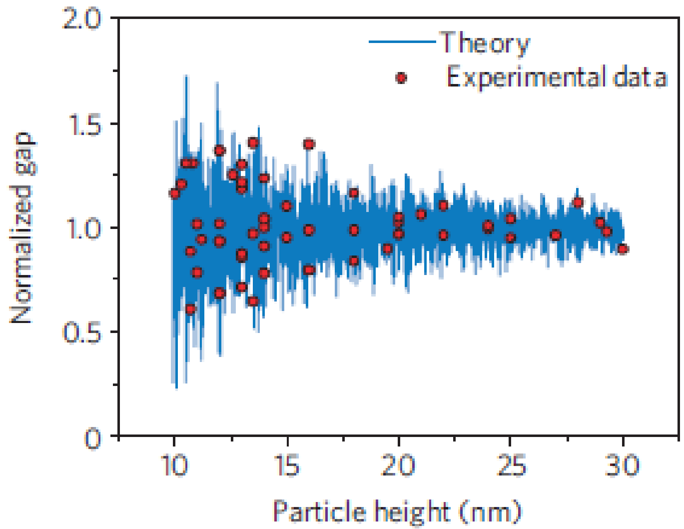
L= 6nm, Neumann, $\delta/\Delta_{0,}$=0.67

L = 8nm, Dirichlet, δ/Δ_0=0.32

L = 10nm, Dirichlet, $\delta/\Delta_{0,}$= 0.08

Single, Isolated Sn nano-grains





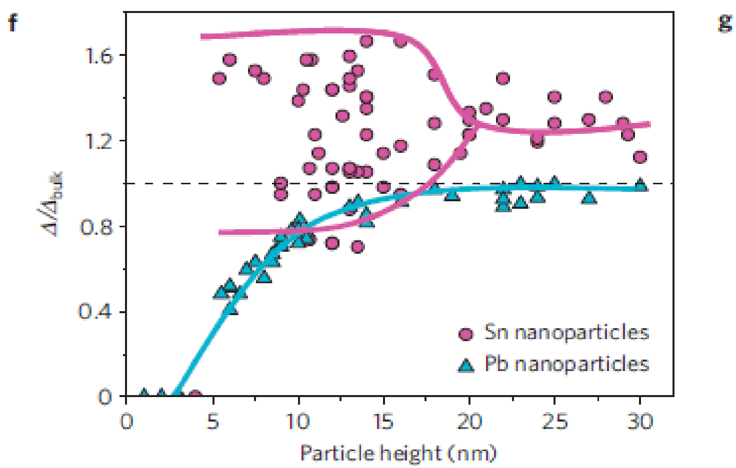
Kern

Bose

R ~ 4-30nm

B closes gap

Almost hemispherical

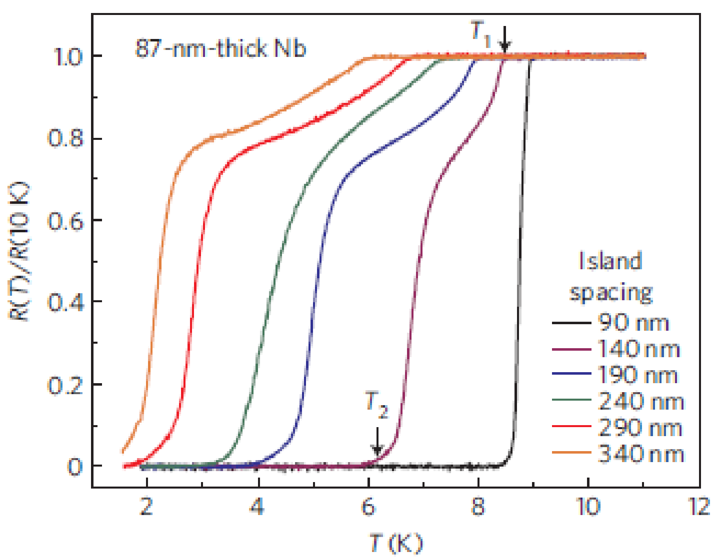


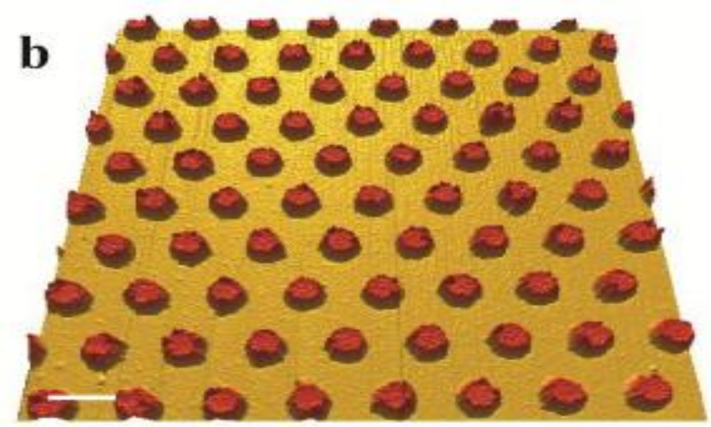
S. Bose, AGG et al., Nature Materials 9, 550 (2010)

True

long-range order in nanograins?

Josephson junctions





Mason, Goldbart et al, Nature Physics 8 59 (2012)

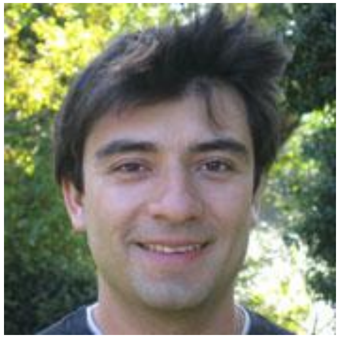
James Mayoh and AGG, in preparation

d = 1

Quasi long-range order

1d + Dissipation = Long range order

Giamarchi, Blatter, Zaikin, Fisher, Lobos..



A. Lobos
Maryland

Why not long-range order in 1d?

M. Tezuka
Kyoto

Phase coherence in 1d by power-law hopping

AGG et al., arXiv:1212.6779

1d Hubbard + power-law hopping

$$\mathcal{H} = -\sum_{l\neq m,\sigma}^{L}\left(t_{lm}\hat{c}^{\dagger}_{l,\sigma}\hat{c}_{m,\sigma} + \text{H.c.}\right) - \mu\sum_{l=1,\sigma}^{L}\left(\hat{n}_{l,\sigma} - \frac{1}{2}\right)$$

$$-|U|\sum_{l=1}^{L}\left(\hat{n}_{l,\uparrow} - \frac{1}{2}\right)\left(\hat{n}_{l,\downarrow} - \frac{1}{2}\right), \quad t_{lm} = t/|l-m|^{\alpha}$$

$$|U| \gg t$$

$$\mathcal{H}_{\text{eff}} = \frac{|U|\,L}{4} - \mu\sum_{l}\left(\hat{n}_l - 1\right)$$

$$+\frac{4t^2}{|U|}\sum_{l\neq m}\left[\frac{\left(\hat{n}_l - 1\right)\hat{n}_m - \hat{\Delta}^{\dagger}_{l}\hat{\Delta}_m}{|l-m|^{2\alpha}} + \text{H.c.}\right]$$

Previous

Short-range hopping	+	U<0 Clean	SC over CDW Korepin
Power-law hopping	+	U=0 Disorder	$d_{eff} = \frac{2}{2\alpha - 1}$ Mirlin, Fyodorov
Power-law hopping	+	U<0 Disorder	Thermalization without time scale Rigol, Relano, AGG
Power-law $\alpha = 1$	+	U<0 μ=1/2 Flux	Haldane Shastry Gebhard and Ruckenstein

Bosonization

$$\rho\left(x\right)=\left[\rho_{0}-\frac{\nabla\phi(x)}{\pi}\right]\sum_{p}e^{2ip(\pi\rho_{0}x-\phi(x))}$$

$$\Delta\left(x\right)=\rho_{0}e^{-i\theta(x)}\sum_{p}e^{2ip(\pi\rho_{0}x-\phi(x))},$$

$$\left[\nabla\phi\left(x\right),\theta\left(y\right)\right]=i\pi\delta\left(x-y\right)$$

Phase

$$\langle\Delta(x)\rangle=\langle\hat{c}_{x,\uparrow}^{\dagger}\hat{c}_{x,\downarrow}^{\dagger}\rangle\propto\langle e^{-i\theta(x)}\rangle$$

Density

$$\delta\rho\left(x\right)\simeq-\nabla\phi\left(x\right)/\pi$$

$$\mathcal{H}_{\text{eff}}=\int dx\left[\tilde{\mu}\frac{\nabla\phi\left(x\right)}{\pi}+\frac{uK}{2\pi}\left(\nabla\theta\left(x\right)\right)^{2}+\frac{u}{2\pi K}\left(\nabla\phi\left(x\right)\right)^{2}\right]$$

$$-g\frac{\pi u\rho_{0}^{2}}{4Ka^{1-2\alpha}}\int_{|x-x'|>a}dxdx'\frac{\cos\left[\theta\left(x\right)-\theta\left(x'\right)\right]}{|x-x'|^{2\alpha}}.$$

Self-consistent harmonic approximation (SCHA)

$$S_0 = \frac{1}{2\beta L} \sum_{\mathbf{q}} g_0^{-1}(\mathbf{q})\, \theta^*_{\mathbf{q}} \theta_{\mathbf{q}}$$

$$F_{\mathbf{var}} = F_0 + T \langle S - S_0 \rangle_0$$

$$g_0^{-1}(\mathbf{q}) = \frac{K}{\pi u}\omega_m^2 + \frac{uK}{\pi}k^2 + \frac{2\pi u \rho_0^2}{K a^{2-2\alpha}} \int_a^L dr \frac{1-\cos kr}{r^{2\alpha}} \times \exp\left[-\frac{1}{\beta L}\sum_{\mathbf{q}'} (1-\cos k' r)\, g_0(\mathbf{q}')\right]$$

$$g_0^{-1}(\mathbf{q}) = \frac{K}{\pi u}\omega_m^2 + \frac{uK}{\pi}k^2 + \eta |k|^{2\alpha-1}$$

$\alpha < 3/2$	Long-range order
$\alpha > 3/2$	Quasi long-range order

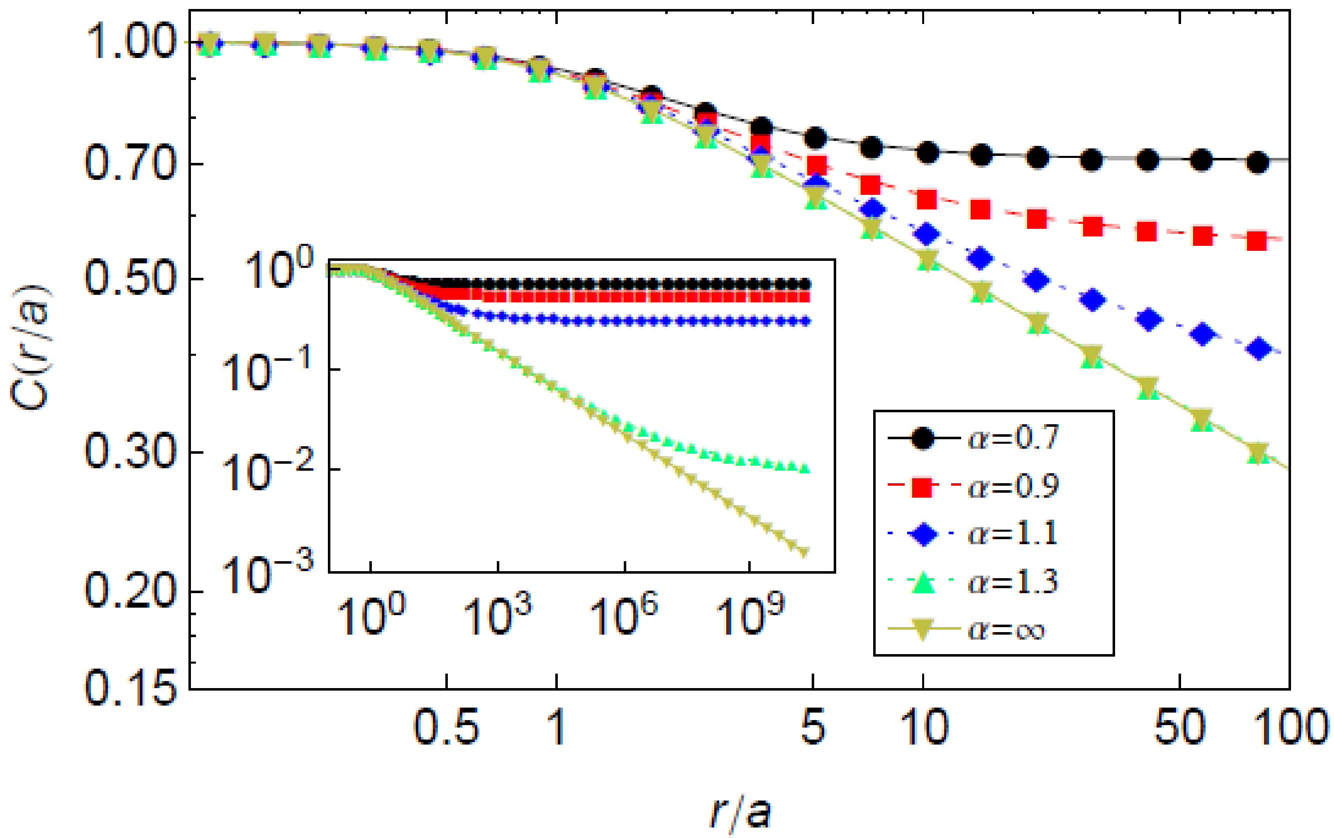
$$\langle \Delta(x_i) \rangle \sim \left\langle e^{i\theta(x_i)} \right\rangle \neq 0$$

Phase slips suppressed

$d = 1$ BUT $d_{eff} = 2/(2\alpha - 1)$

$d_{eff} > 1$ Phase coherence

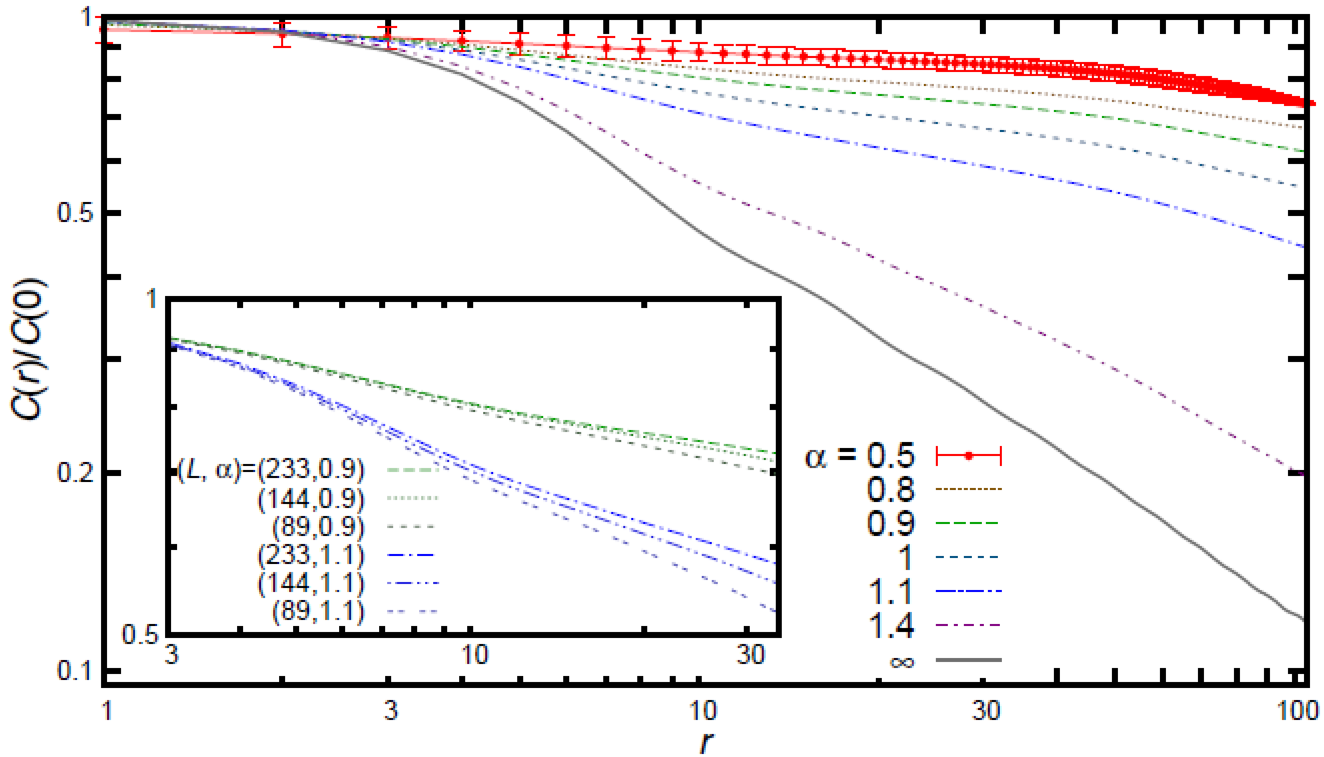
Bosonization:Correlations



$$C\left(r\right) = \left\langle e^{i\theta(r)} e^{-i\theta(0)} \right\rangle_0$$

$$C\left(r\right) \approx e^{-G(0)} \left[1 + A/r^{3/2-\alpha} + \mathcal{O}\left(1/r^{3-2\alpha}\right)\right]$$

Numerical: DMRG



$$C(r) \equiv \frac{1}{L - 2l_0 - r} \sum_{l=l_0+1}^{L-l_0-r} \langle \hat{\Delta}_{l+r} \hat{\Delta}_l^\dagger \rangle$$

$$\hat{\Delta}_l^\dagger \equiv \hat{c}_{l,\uparrow}^\dagger \hat{c}_{l,\downarrow}^\dagger$$

From SC to Quantum magnetism

$$\mathcal{H}_{\rm eff} = \frac{|U|\,L}{4} - \mu \sum_l (\hat{n}_l - 1)$$

$$+ \frac{4t^2}{|U|} \sum_{l \neq m} (-1) \left[\frac{(\hat{n}_l - 1)\,\hat{n}_m - \hat{\Delta}_l^\dagger \hat{\Delta}_m}{|l-m|^{2\alpha}} + \text{H.c.} \right]$$

Anderson Pseudo-spin representation

$$\hat{n}_l \rightarrow \hat{S}_l^z + 1/2$$

$$\hat{\Delta}_l^\dagger \rightarrow \hat{S}_l^+$$

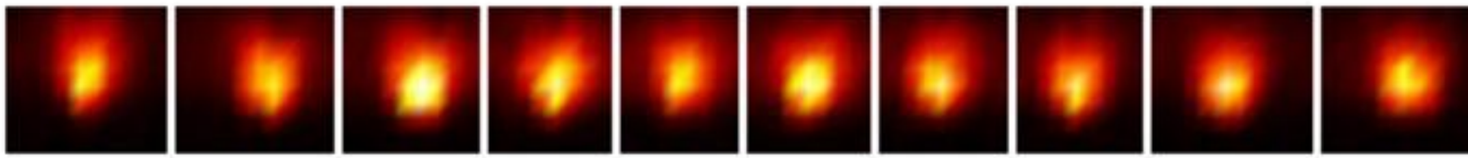
XXZ Spin chain

Heisenberg chain

Variable-Range Interactions in Trapped Ion Quantum Simulators

Islam, Monroe.. 1210.0142, Bollinger, Britton… Nature 484, 489 (2012)

$^{171}Yb^{+}$



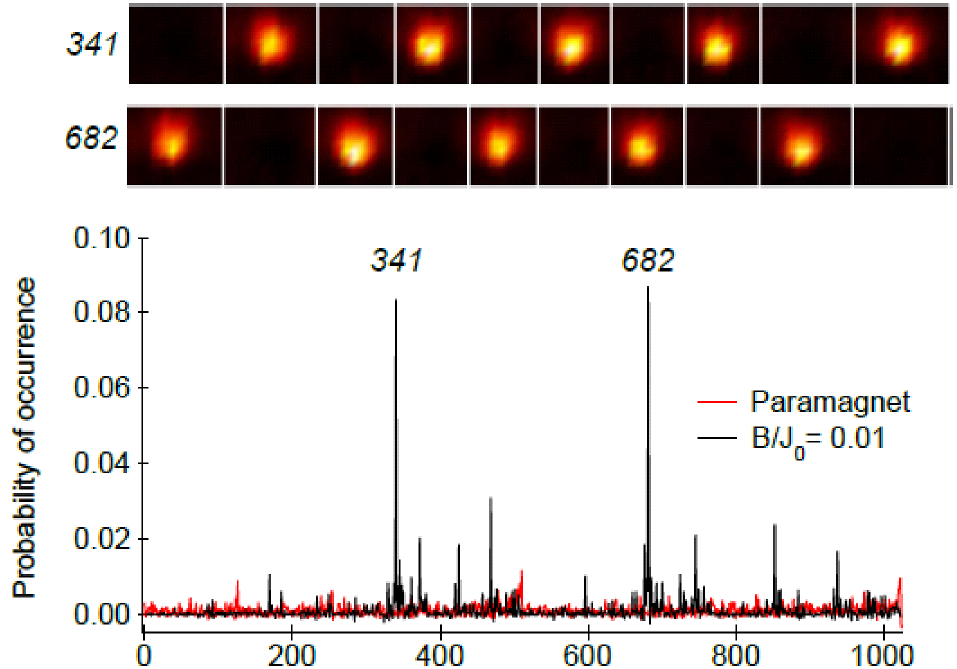
Raman transitions

π/2-shift

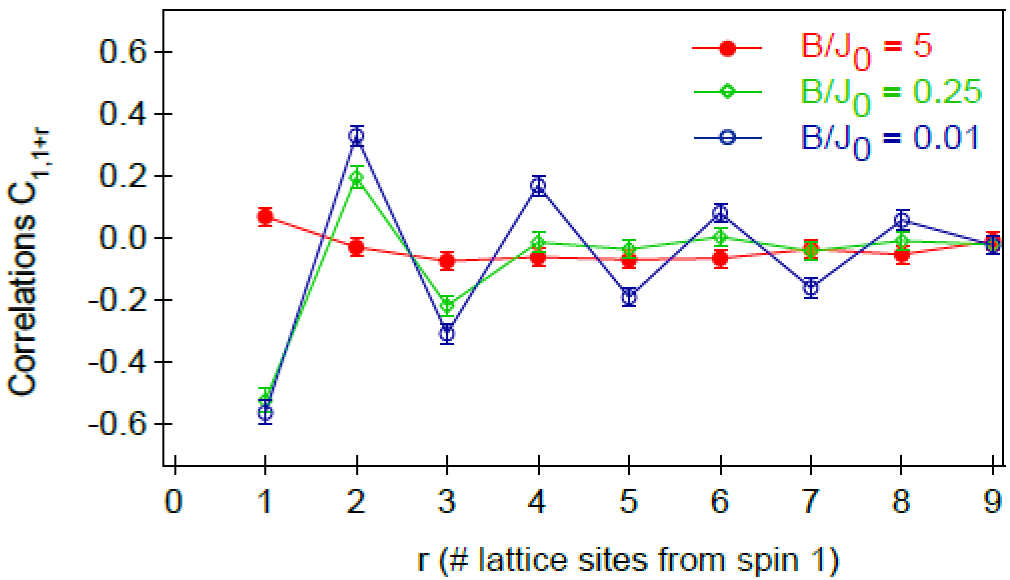
Dipole forces

$$H = \sum_{j<i} J_{ij}\sigma_x^{(i)}\sigma_x^{(j)} - B\sum_i \sigma_y^{(i)} \qquad J_{ij} \approx \frac{J_0}{|i-j|^{\alpha}}$$

$$0 < \alpha < 3$$



$$C_{1,1+r} = \langle \sigma_x^{(1)} \sigma_x^{(1+r)} \rangle - \langle \sigma_x^{(1)} \rangle \langle \sigma_x^{(1+r)} \rangle$$



(anti)Ferromagnetic Transitions

Frustration

Spin Liquids

Quantum Magnetism

Nano meets high

Holographic grains

Finite Size
+
Strong interactions ?

Tough for even conventional superconductors

Way
Santa Barbara

Holographic superconductivity in confined geometries?

Santos
Santa Barbara

AGG, et al., PRB, 86, 064526 (2012)

Holographic principle

Maldacena’s conjecture

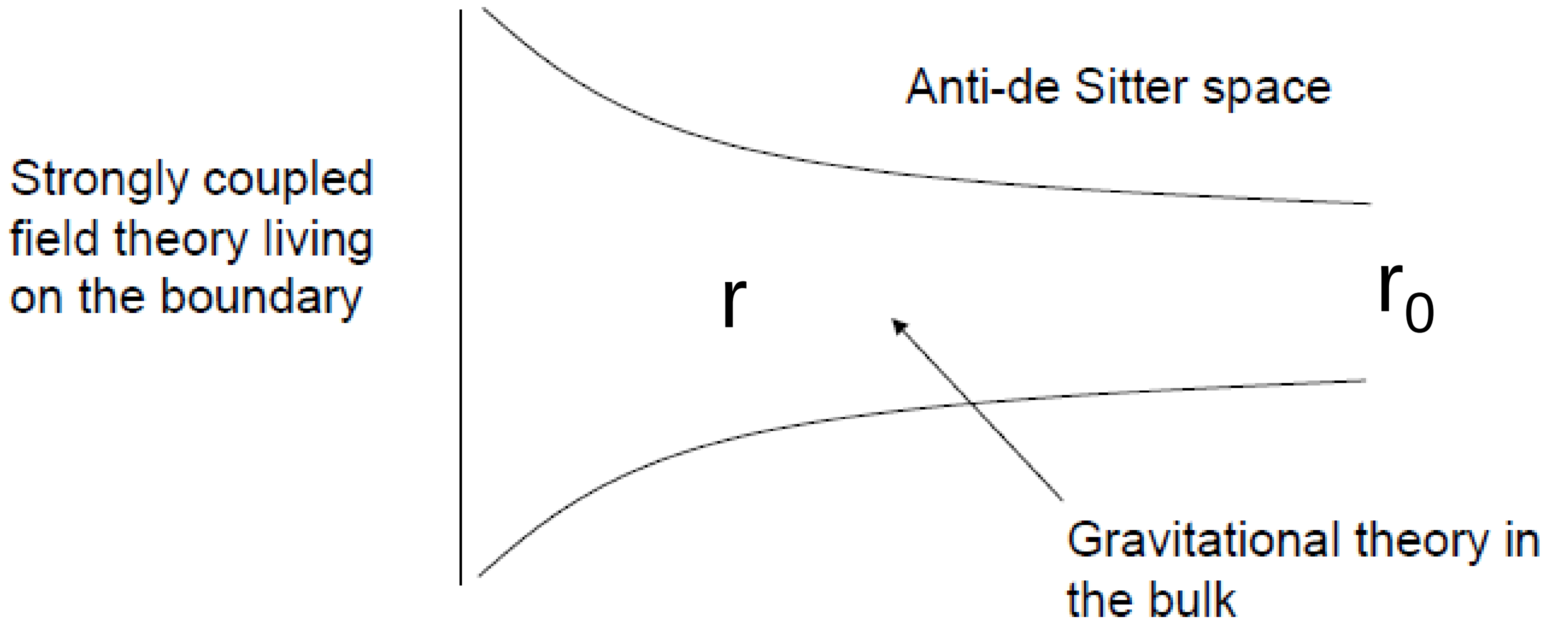
AdS/CFT correspondence

t’Hooft, Susskind, Weinberg, Witten....

Strongly coupled field theory in d		Weakly coupled gravity in d+1
N=4 Super-Yang Mills CFT	⟺	Anti de Sitter space AdS

Extra dimension?

Geometrization of Wilson RG



Holography beyond string theory

2003	QCD Quark gluon plasma	Gubser, Son
2008	Holographic superconductivity	Hartnoll, Herzog, Horowitz
2012	Quantum criticality, non-equilibrium..	Zaanen, Sachdev, Philips

Easy to compute in the gravity dual & Detailed dictionary

An answer looking for a question

H = ?

I do not know

Complex scalar

I know that

Spontaneous breaking U(1) at low T

Finite μ

Simplest dual gravity theory

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D\psi|^2 - m^2 |\psi|^2 \right]$$

$$D = \nabla - iqA$$

ψ ≡ complex scalar

Metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2)$$

$$= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dR^2 + R^2 d\theta^2)$$

$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_0^3}{r^3}\right),$$

Equations of motion:

$$\partial_r^2|\psi| + \frac{1}{r^2 f}\partial_x^2|\psi| + \left(\frac{f'}{f} + \frac{2}{r}\right)\partial_r|\psi| + \frac{1}{f}\left(\frac{A_t^2}{f} - m^2\right)|\psi| = 0$$

$$\partial_r^2 A_t + \frac{1}{r^2 f}\partial_x^2 A_t + \frac{2}{r}\partial_r A_t - \frac{2|\psi|^2}{f}A_t = 0$$

Boundary conditions:

$r = r_0$ **$r \to \infty$**

$A_t = 0$

$$|\psi| = \frac{\psi^{(1)}}{r} + \frac{\psi^{(2)}}{r^2} + O\left(\frac{1}{r^3}\right)$$

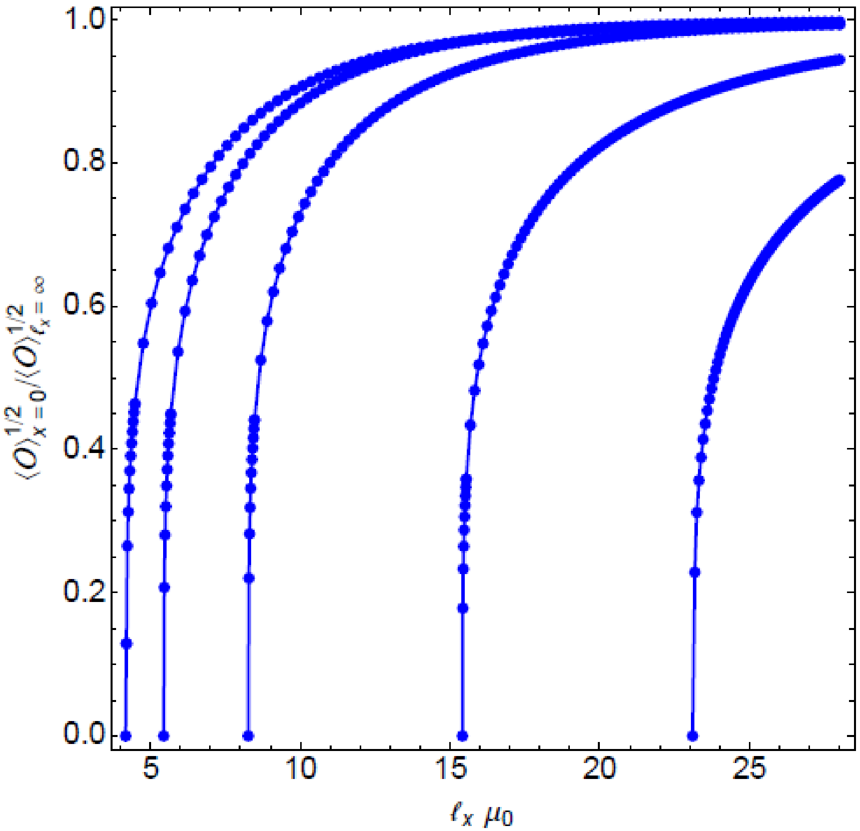
$$A_t = \mu - \frac{\rho}{r} + O\left(\frac{1}{r^2}\right)$$

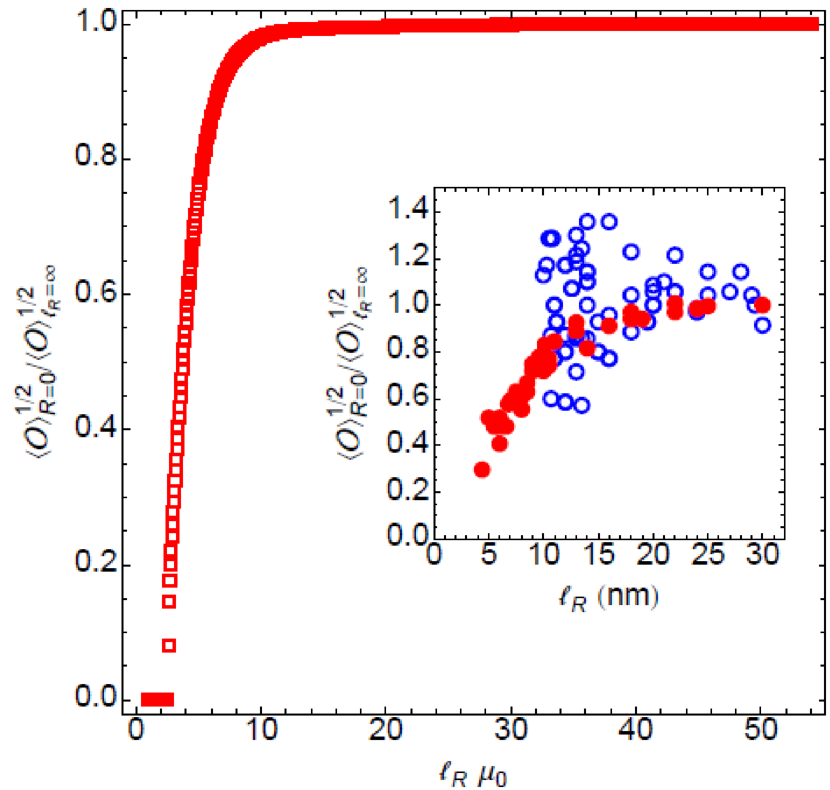
How small?

$$\mu(x) = \mu_0\left[\frac{1 - \epsilon + \epsilon\cosh\left(\frac{2x}{\sigma}\right) + \cosh\left(\frac{\ell_x}{\sigma}\right)}{\cosh\left(\frac{2x}{\sigma}\right) + \cosh\left(\frac{\ell_x}{\sigma}\right)}\right]$$

Dictionary:

$$\langle\mathcal{O}\rangle = \sqrt{2}\psi^{(2)}$$

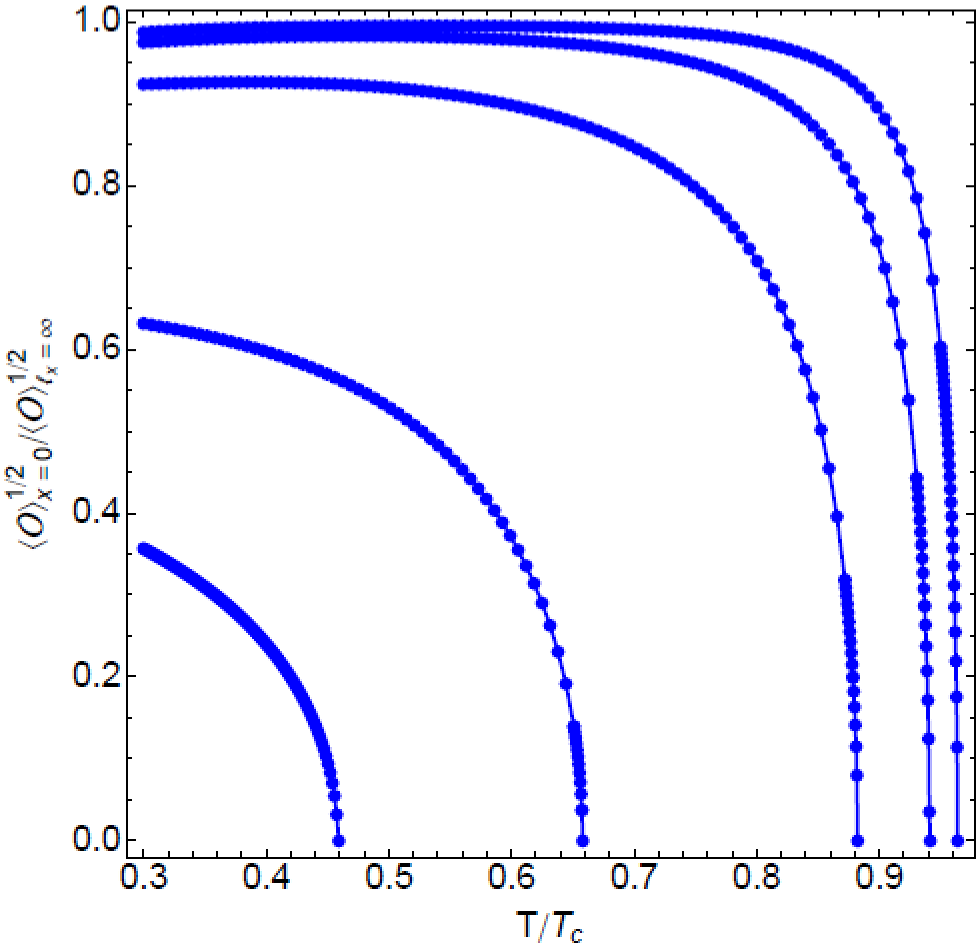




“Superconductivity” only for $l < l_c$

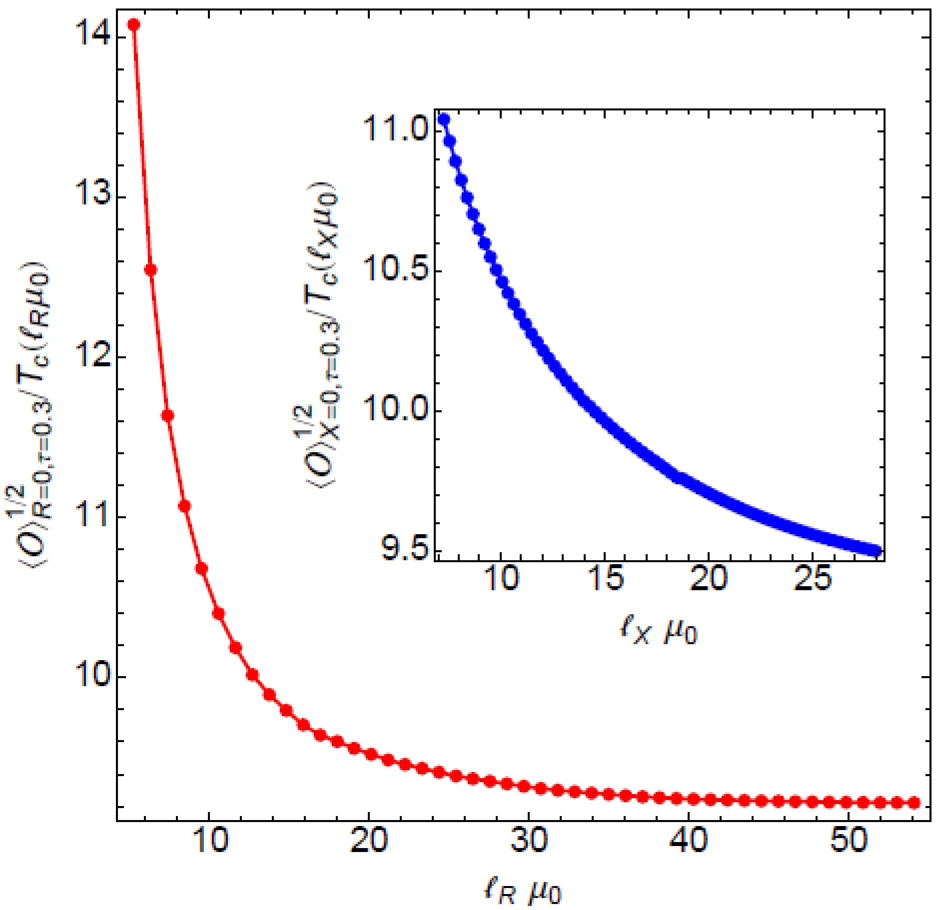
Mean field behavior

Fluctuations?



No thermal fluctuations

Large N artefact

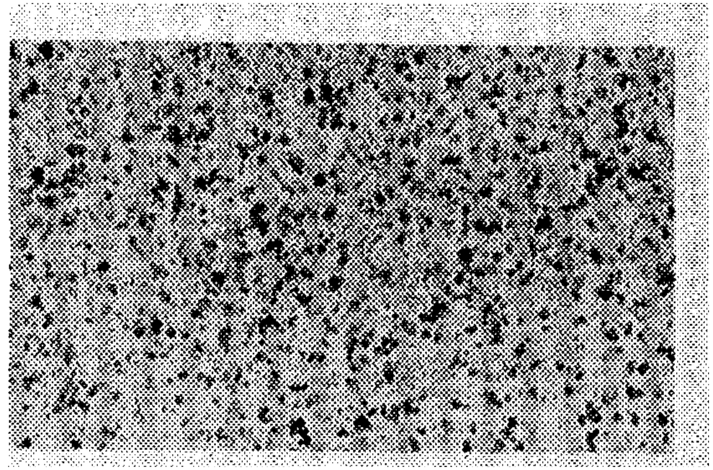


Interactions depend on system size!

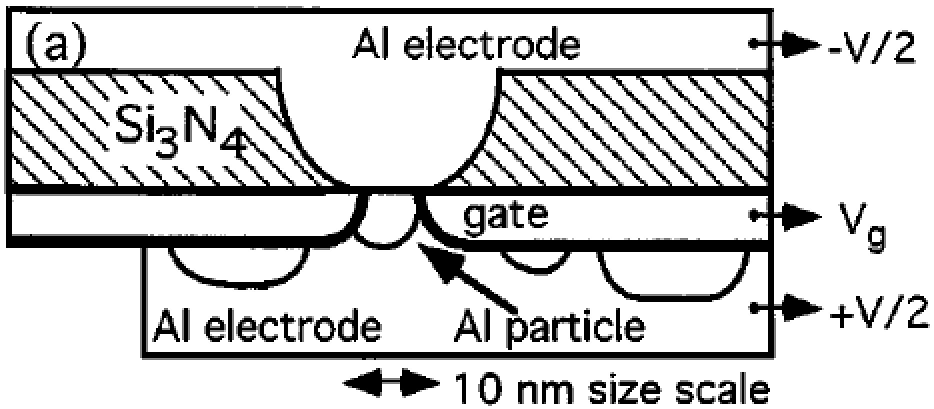
PRB, 86, 064526 (2012)

+Experimental control

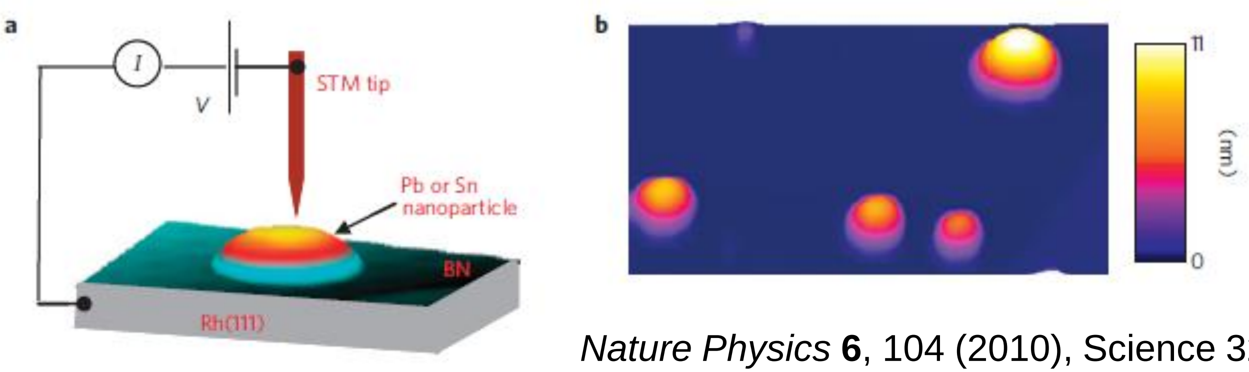
+Predictive power



1966



1995



Now

Nature Physics **6**, 104 (2010), Science 324, 1314 (2009).

Next

Theory

Heterostructures
Collections of grains

Topology

Non-equilibrium

Experiments

Control on high T_c heterostructures

Control on grains arrangements

Substantial enhancement of T_c

THANKS!