The non-equilibrium birth of a superfluid

Antonio M. García-García

http://www.tcm.phy.cam.ac.uk/~amg73/

arXiv:1407.1862

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FCT
Fundação para a Ciência e a Tecnologia
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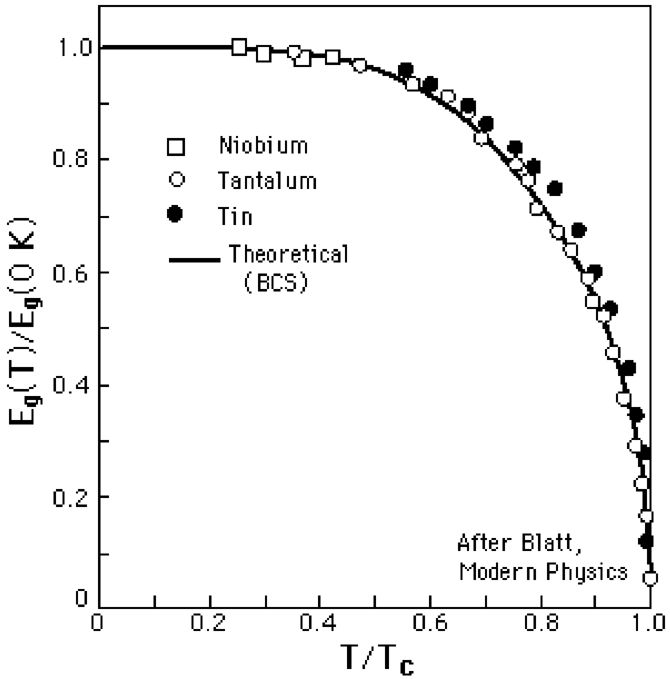
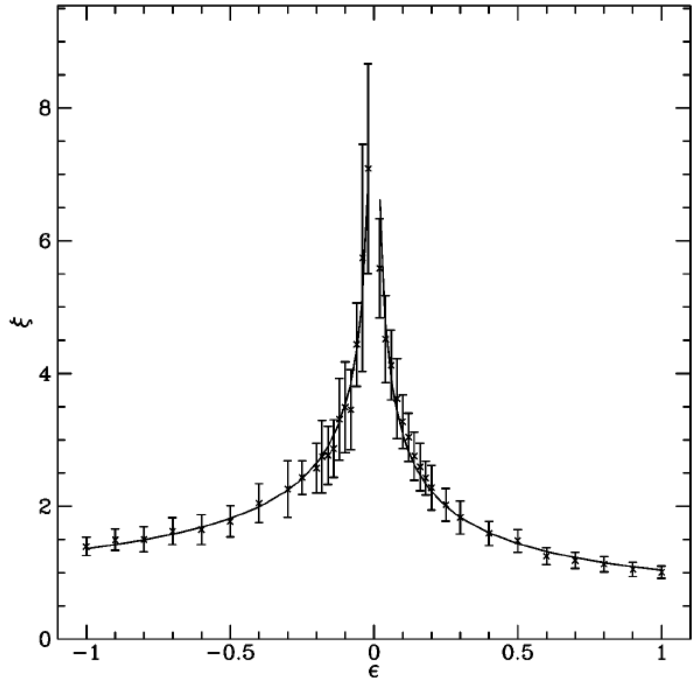
MARIE CURIE ACTIONS

EPSRC
Engineering and Physical Sciences Research Council

Second order phase transitions



$$\epsilon = (T - T_c)/T_c$$



$$\langle \psi \rangle \neq 0 \quad T < T_c$$

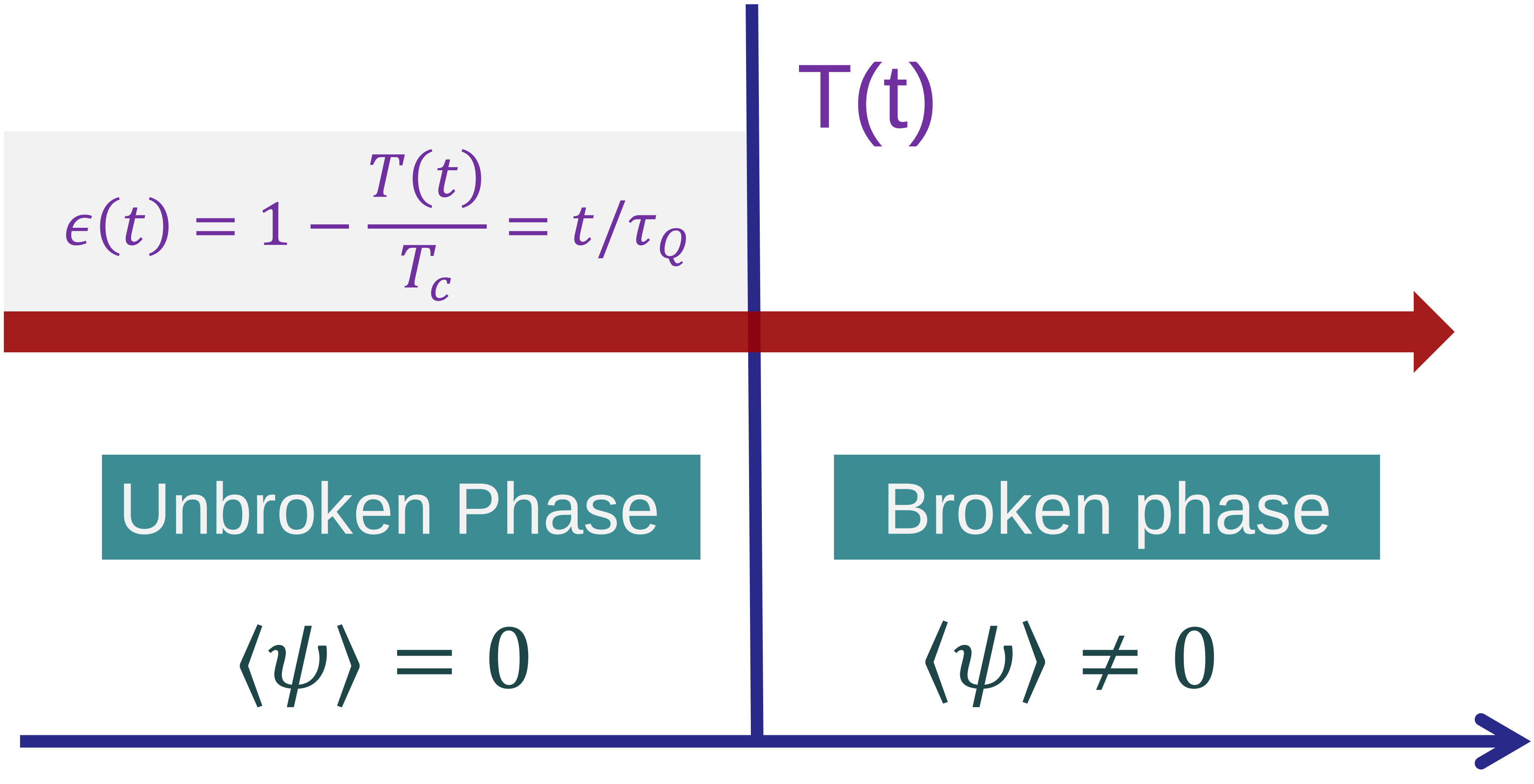
$$\langle \psi \rangle = 0 \quad T > T_c$$

$$\tau_{eq} = \tau_0 |\epsilon|^{-\nu z}$$

$$\xi_{eq} = \xi_0 |\epsilon|^{-\nu}$$

Drive from $\langle \psi \rangle = 0$ to $\langle \psi \rangle \neq 0$?

Dynamical phase transitions



$$\langle \psi \rangle = \Delta(x,t) e^{i\theta(x,t)} \; ?$$

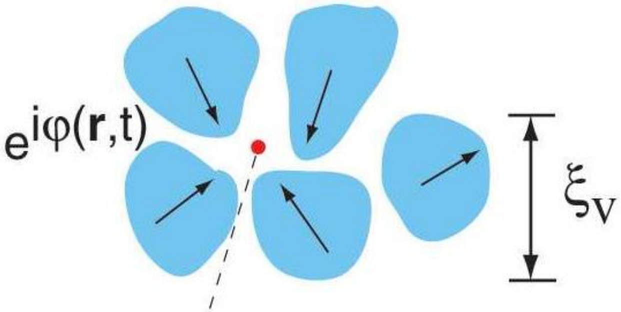
Kibble

J. Phys. A: Math. Gen. 9: 1387. (1976)

Causality

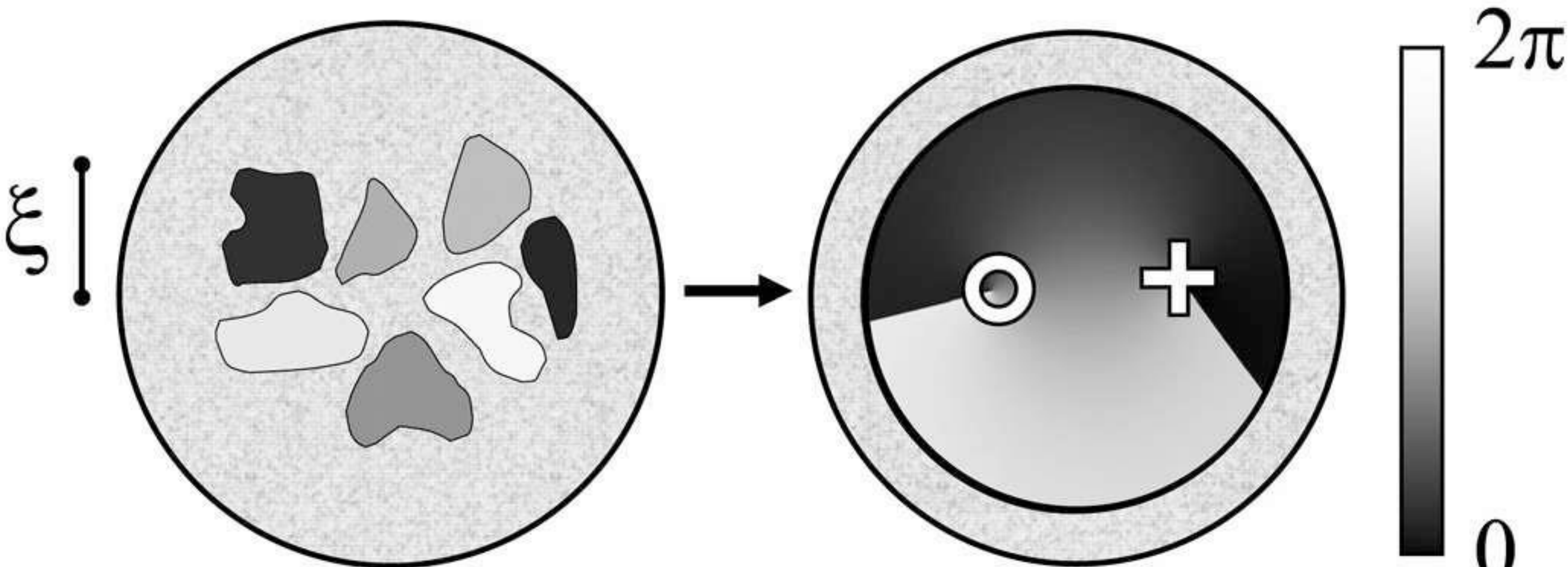
Vortices in the sky

Cosmic strings



Krusius, 2006

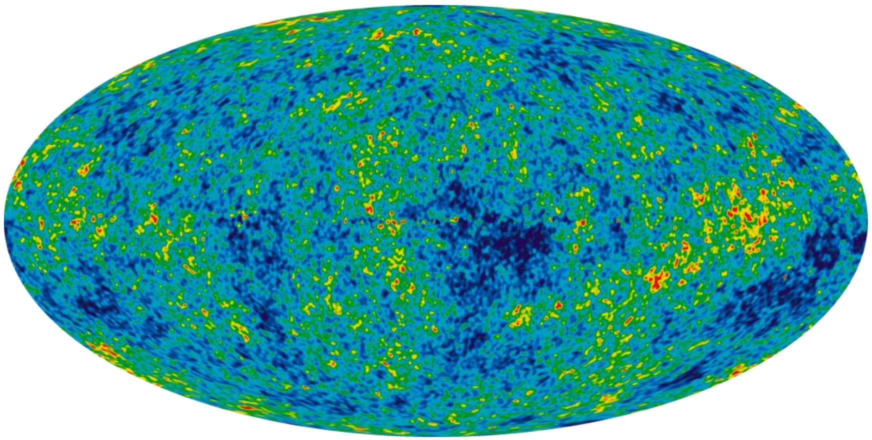
Generation of Structure



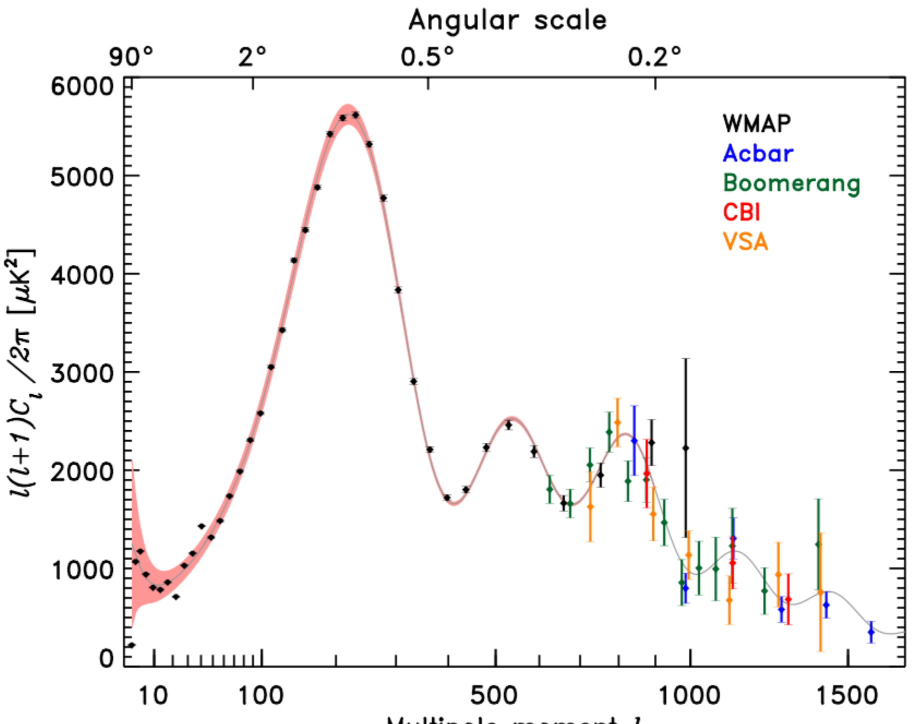
Weyler, Nature 2008

No evidence so far !

CMB, galaxy distributions…



NASA/WMAP



Cosmological experiments in superfluid helium?

W. H. Zurek

Theoretical Astrophysics, Los Alamos National Laboratory,
Los Alamos, New Mexico 87545, USA

Nature 317, 505 (1985)

Doable for ^{4}He!!

Zurek

$T \approx T_c$

2^{nd} order

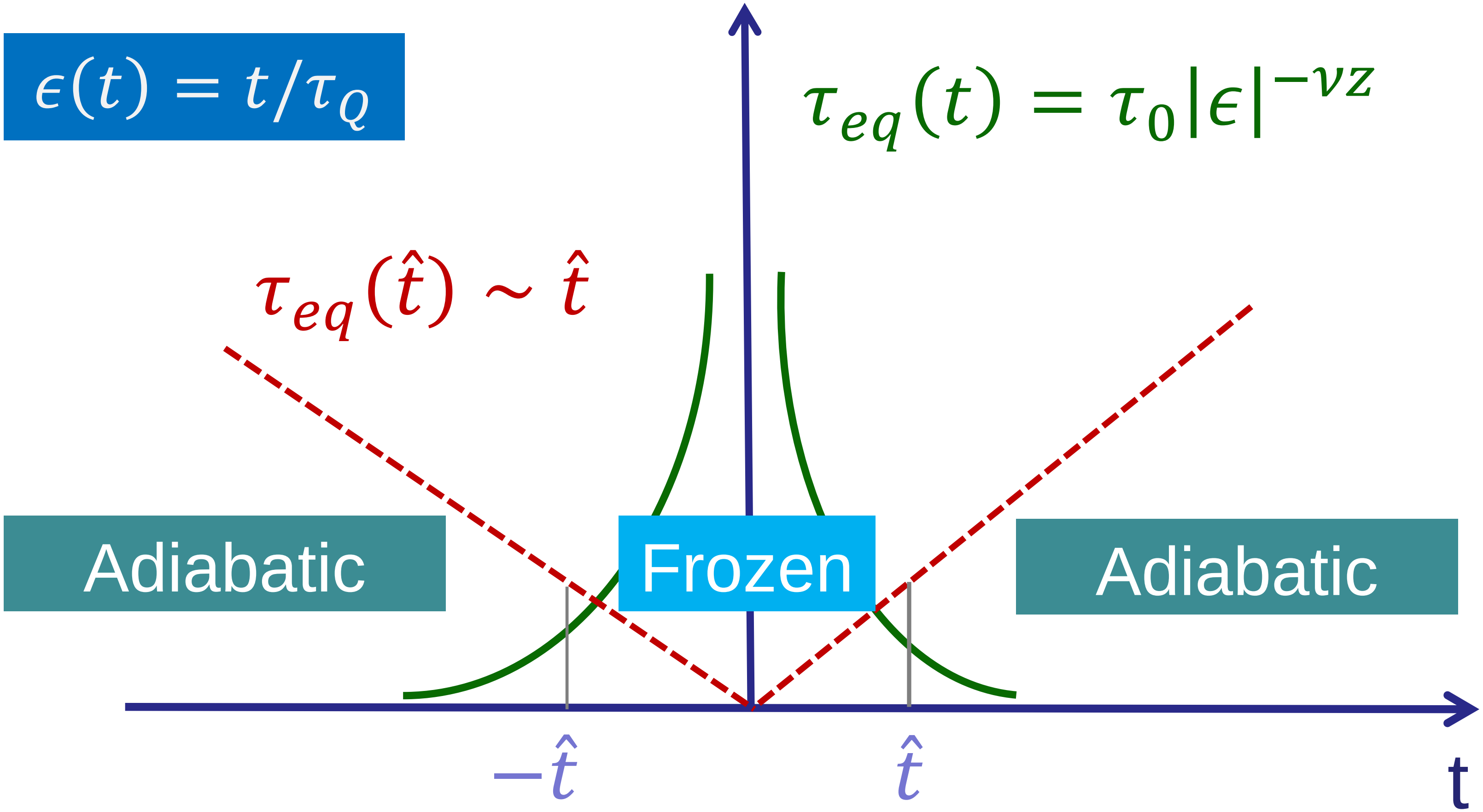
→

Scaling

$\tau(T_c) = \infty$

$$\epsilon(t) = 1 - \frac{T(t)}{T_c} = t/\tau_Q$$

$t = -\hat{t} \equiv -t_{freeze}$	Non adiabatic evolution
$t = \hat{t} \equiv t_{freeze}$	Defect generation!



$$\hat{\xi} = \xi_0|\hat{\epsilon}|^{-\nu} = \xi_0\left(\tau_Q/\tau_0\right)^{\nu/(1+\nu z)}$$

Kibble-Zurek mechanism

$$\rho \sim \hat{\xi}^{-d} \sim \tau_Q^{-d\nu/(1+\nu z)}$$

Generation of defects in superfluid ^{4}He as an analogue of the formation of cosmic strings

P. C. Hendry*, N. S. Lawson*, R. A. M. Lee*, P. V. E. McClintock* & C. D. H. Williams†

* School of Physics and Materials, Lancaster University, Lancaster LA1 4YB, UK
† Department of Physics, University of Exeter, Exeter EX4 4QL, UK

NATURE · VOL 368 · 24 MARCH 1994

Transient attenuation of second sound amplitude

But vortices induced by stirring up!

VOLUME 81, NUMBER 17 PHYSICAL REVIEW LETTERS 26 OCTOBER 1998

Nonappearance of Vortices in Fast Mechanical Expansions of Liquid ^{4}He through the Lambda Transition

M. E. Dodd,[1] P. C. Hendry,[1] N. S. Lawson,[1] P. V. E. McClintock,[1] and C. D. H. Williams[2]
[1]*Department of Physics, Lancaster University, Lancaster, LA1 4YB, United Kingdom*
[2]*Department of Physics, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom*

No vortices in ^{4}He!!

G. Karra, R. J. Rivers, PRL. 81, 3707 (1998)

Vortex formation in neutron-irradiated superfluid ^3He as an analogue of cosmological defect formation

Ruutu, Nature 382, 334-336 (1996)

OK

Laboratory simulation of cosmic string formation in the early Universe using superfluid ^3He

C. Bäuerle et al. Nature 382, 332 (1996)

Thin SC films, nematic liquid crystal..

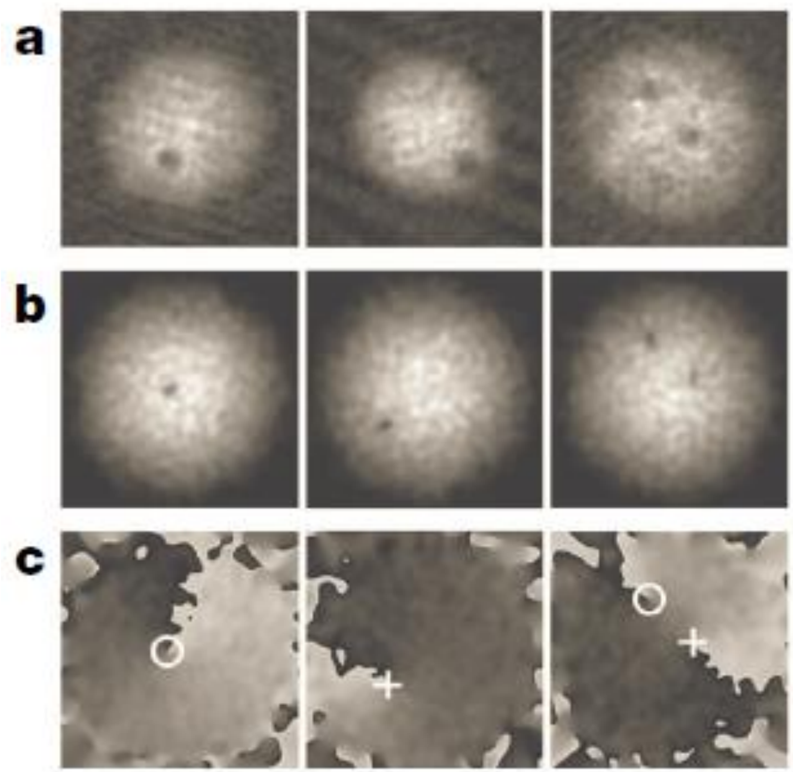
?

nature Vol 455|16 October 2008|doi:10.1038/nature07334

LETTERS

Spontaneous vortices in the formation of Bose–Einstein condensates

Chad N. Weiler[1], Tyler W. Neely[1], David R. Scherer[1], Ashton S. Bradley[2]†, Matthew J. Davis[2] & Brian P. Anderson[1]



nature COMMUNICATIONS

ARTICLE

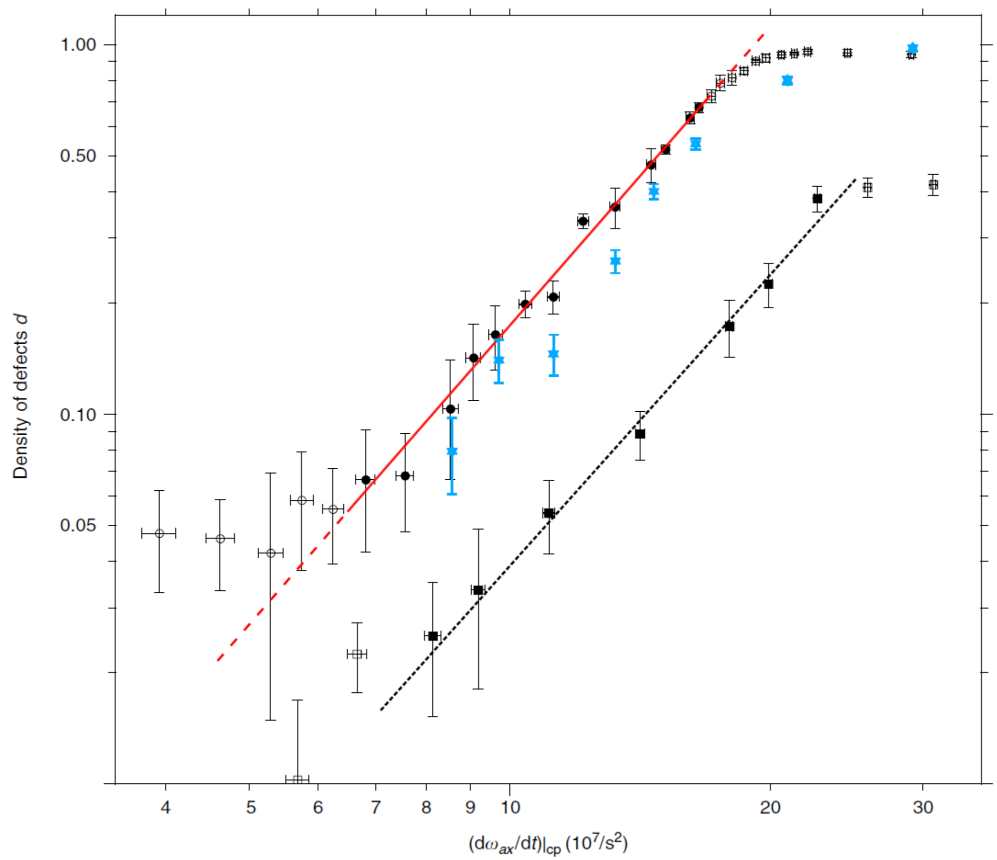
Received 25 Mar 2013 | Accepted 11 Jul 2013 | Published 7 Aug 2013 DOI: 10.1038/ncomms3290

Observation of the Kibble-Zurek scaling law for defect formation in ion crystals

S. Ulm[1], J. Roßnagel[1], G. Jacob[1], C. Degünther[1], S.T. Dawkins[1], U.G. Poschinger[1], R. Nigmatullin[2,3], A. Retzker[4], M.B. Plenio[2,3], F. Schmidt-Kaler[1] & K. Singer[1]

KZ scaling with the quench speed

Too few defects



Extension to quantum phase transitions

Zurek, Zoller, et al, "Dynamics of a quantum phase transition." , PRL 95.10 (2005): 105701.

Analytical demonstration of KZ scaling in 1d Ising chain in transverse field

Dziarmaga "Dynamics of a quantum phase transition: Exact solution of the quantum Ising model." PRL 95.24 (2005): 245701.

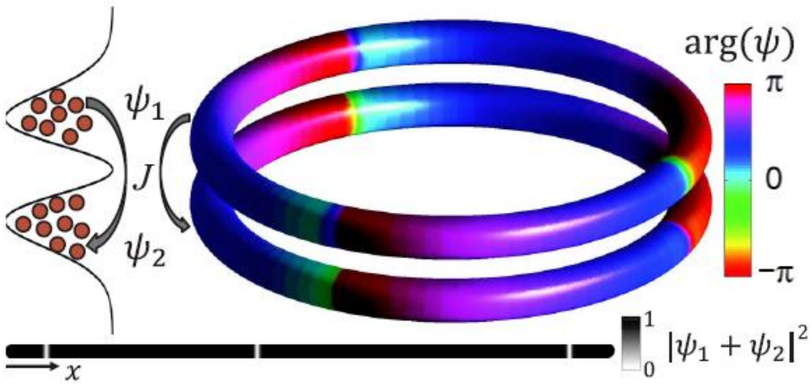
Calculation of correlation functions

Kibble-Zurek problem: Universality and the scaling limit PRB 86, 064304, (2012), Gubser, Sondhi et al.

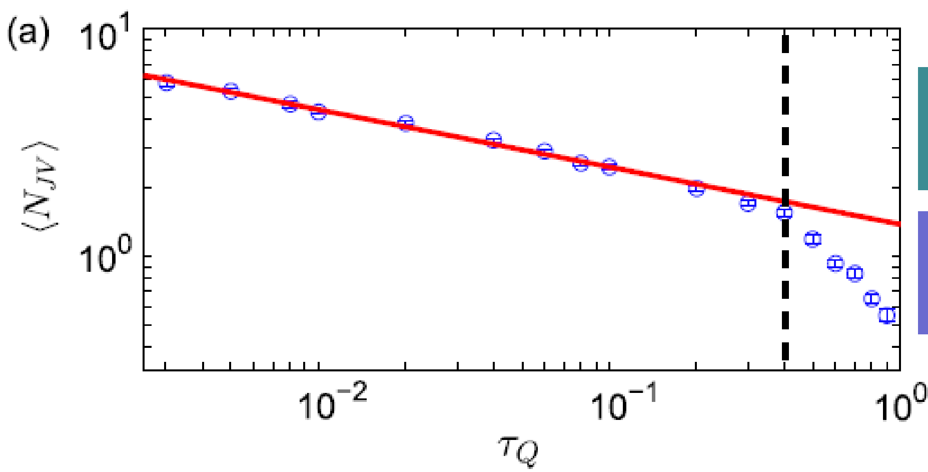
PRL 110, 215302 (2013) PHYSICAL REVIEW LETTERS week ending 24 MAY 2013

Kibble-Zurek Scaling and its Breakdown for Spontaneous Generation of Josephson Vortices in Bose-Einstein Condensates

Shih-Wei Su,[1] Shih-Chuan Gou,[2] Ashton Bradley,[3] Oleksandr Fialko,[4] and Joachim Brand[4]



Stochastic Gross-Pitaesvkii



Breaking of KZ scaling

Too few vortices !

Issues with KZ

$$\rho_{\mathrm{KZ}} \sim 1/\xi_{\mathrm{freeze}}^{d-D} \sim \tau_Q^{(d-D)\nu/(1+\nu z)}$$

Too many vortices

When does KZ scaling stop?

Fast quenches?

Can be t_{freeze} be truly relevant ?

Dynamic does not have to be adiabatic at t_{freeze}

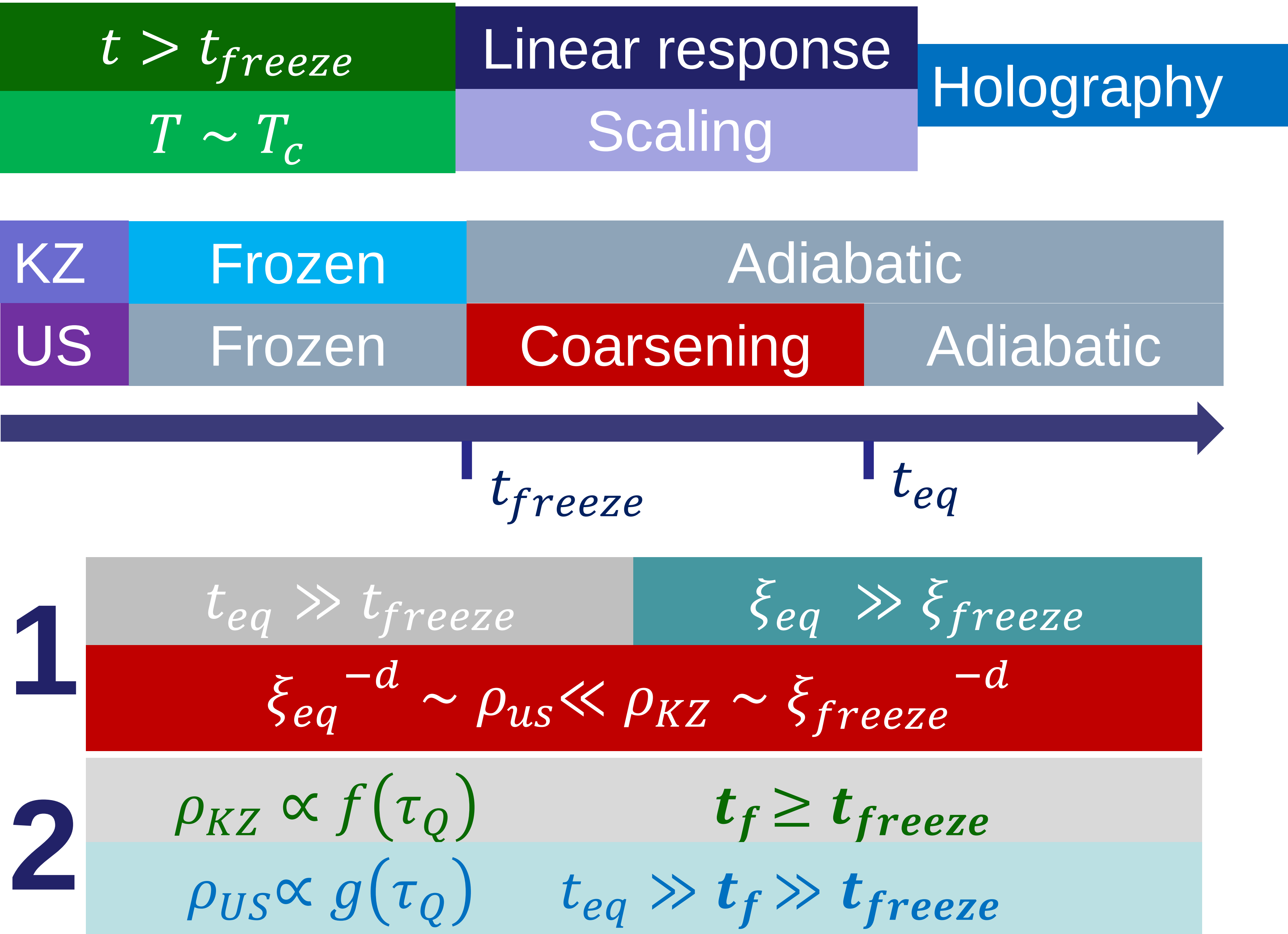
No defects without a well formed condensate

Another scale in the problem

$t > t_{freeze}$ is relevant

arXiv:1407.1862

Chesler, AGG, Liu



$t > t_{freeze}$
$T \sim T_c$
Linear response
Scaling
Holography
KZ
Frozen
Adiabatic
US
Frozen
Coarsening
Adiabatic
t_{freeze}
t_{eq}
1
$t_{eq} \gg t_{freeze}$
$\xi_{eq} \gg \xi_{freeze}$
${\xi_{eq}}^{-d} \sim \rho_{us} \ll \rho_{KZ} \sim {\xi_{freeze}}^{-d}$
2
$\rho_{KZ} \propto f(\tau_Q)$
$t_f \geq t_{freeze}$
$\rho_{US} \propto g(\tau_Q)$
$t_{eq} \gg t_f \gg t_{freeze}$

Non adiabatic growth after t_{freeze}

$$C(t,\boldsymbol{r}) \equiv \langle \psi^*(t,\boldsymbol{x}+\boldsymbol{r})\psi(t,\boldsymbol{x})\rangle$$

$$\psi(t,\boldsymbol{q}) = \int dt' G_{\mathrm{R}}(t,t',q)\varphi(t,\boldsymbol{q})$$

$$\langle\varphi^*(t,\boldsymbol{x})\varphi(t',\boldsymbol{x}')\rangle = \zeta\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}')$$

$$G_R(t,t',q) = \theta(t-t')H(q)e^{-i\int_{t'}^{t} dt''\mathfrak{w}_0(\epsilon(t''),q)}$$

$$C(t,q) = \int dt'\,\zeta|G_R(t,t',q)|^2$$

Linear response $t > t_{freeze}$

$|\partial_t \log \mathfrak{w}_0| < |\mathfrak{w}_0|$

$$C(t,q) = \int_{t_{\rm freeze}}^{t} dt' \zeta |H(q)|^2 e^{2\int_{t'}^{t} dt'' \mathrm{Im}\, \mathfrak{w}_0(\epsilon(t''),q)} + \cdots$$

$$\mathfrak{w}_0(\epsilon, q) = \epsilon^{z\nu} h(q\epsilon^{-\nu})$$

$$\mathrm{Im}\, \mathfrak{w}_0 = -a\epsilon^{(z-2)\nu} q^2 + b\epsilon^{z\nu} + \ldots,$$

$$\mathrm{Im}\, \mathfrak{w}_0 > 0$$

Unstable Modes

$q_{max} \sim \epsilon(t)^{\nu}$

Growth of

$\langle \psi(t) \rangle \ t > t_{freeze}$

Protocol

$$\epsilon(t) = t/\tau_Q$$

$$t \in (t_i, t_f)$$

$$t_i = (1 - T_i/T_c)\tau_Q < 0$$

$$t_f = (1 - T_f/T_c)\tau_Q > 0$$

Slow quenches

$t_f \geq t_{eq}$

$t > t_{freeze}$

Correlation length increases

$$C(t,r) \sim |\psi|^2(t) e^{-\frac{r^2}{2\ell_{co}^2(t)}}, \quad \bar{t} \equiv \frac{t}{t_{freeze}}$$

$$\ell_{co}(\bar{t}) = a_3 \xi_{freeze} \bar{t}^{\frac{1+(z-2)\nu}{2}}$$

Condensate growth

$$|\psi|^2(t) \sim \tilde{\varepsilon}(t) e^{a_2 \bar{t}^{1+z\nu}}$$

$$\tilde{\varepsilon}(t) \equiv \zeta t_{freeze} \ell_{co}^{-d}(t)$$

Adiabatic evolution

$t = t_{eq} \gg t_{freeze}$

$$|\psi|^2(t = t_{eq}) \sim |\psi|^2_{eq}(\epsilon(t_{eq}))$$

 # Defects

$$\rho(t_{eq}) \sim 1/\ell_{co}^{d-D}(t_{eq}) \sim \left[\log(\zeta^{-1}\tau_Q^{\Lambda})\right]^{-\frac{(d-D)(1+(z-2)\nu)}{2(1+z\nu)}} \rho_{KZ}$$

Fast quenches

$$t_f \ll t_{eq}$$

$$q_{max}(T_f) = \epsilon(t_f)^{d\nu}$$

$t > t_{freeze}$

$$C(t,r) = |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}$$

Exponential growth

$$|\psi|^2(t) \sim \epsilon_f^{(d-z)\nu} \zeta \exp\left[2b(t - t_{\rm freeze})\epsilon_f^{\nu z}\right]$$

$$\ell_{\rm co}^2(t) = 4a(t - t_{\rm freeze})\epsilon_f^{\nu(z-2)}$$

Number of defects

$$\rho \sim \begin{cases} \epsilon_f^{(d-D)\nu} & R_f \lesssim O(1) \\ \epsilon_f^{(d-D)\nu} \log^{-\frac{d-D}{2}} R_f & R_f \gg 1 \end{cases} \qquad R_f \equiv \frac{\epsilon_f^{2\beta}}{\zeta \epsilon_f^{(d-z)\nu}}$$

$$\Lambda = (2 - \eta - z)\nu \qquad \epsilon_f \equiv \frac{T_c - T_f}{T_c}$$

Predictions

Fast growth $|\langle\psi(t)\rangle|^2$

$t > t_{freeze}$

# of vortices for fast and slow quenches

$$R \equiv \frac{\tau_Q^{-\frac{2\beta}{1+\nu z}}}{\varepsilon t_{\rm freeze}} \sim \zeta^{-1}\tau_Q^{\frac{\Lambda}{1+\nu z}} \gg 1$$

$$\Lambda \equiv (d-z)\nu - 2\beta$$

$$\frac{t_{\rm eq}}{t_{\rm freeze}} \sim (\log R)^{\frac{1}{1+\nu z}} .$$

$$\frac{\ell_{\rm co}(t_{\rm eq})}{\xi_{\rm freeze}} \sim (\log R)^{\frac{1+(z-2)\nu}{2(1+z\nu)}} .$$

$$\ell_{co}(t_{eq}) \equiv \xi_{eq}$$

Defects only at

$t_{eq} \gg t_{freeze}$

$\rho_{US} \ll \rho_{KZ}$

Breaking of scaling

$t_{freeze} \ll t_f \ll t_{eq}$

KZ $t_f < t_{freeze}$

Holography?

Defects survive large N limit

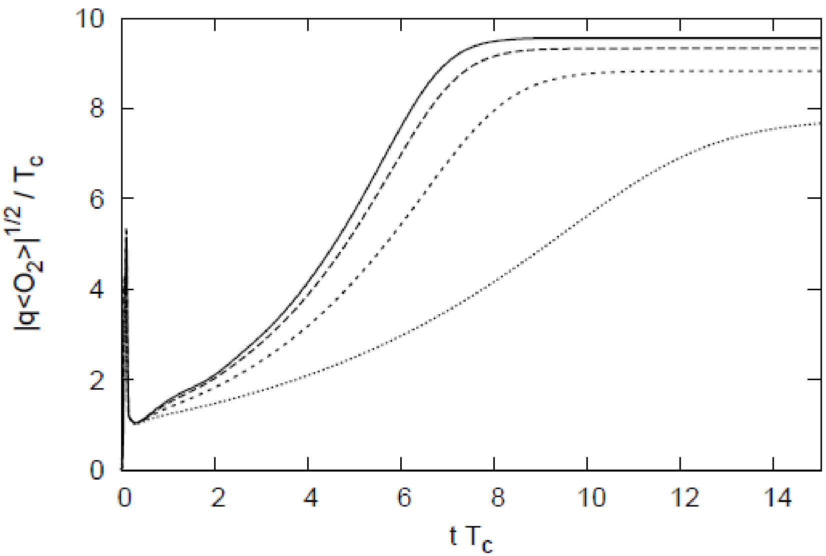
Universality

Real time

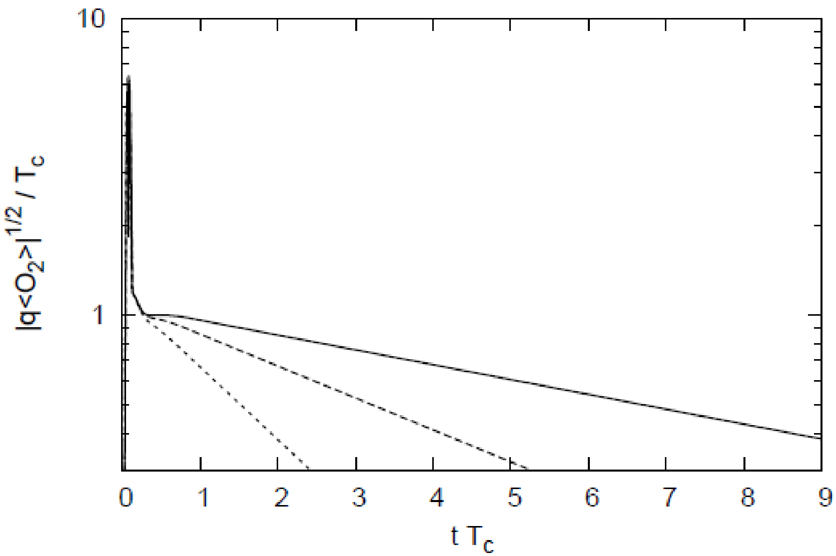
$\langle O_2(t) \rangle$

Backreaction

Murata, et al., arXiv:1005.0633



(a) $T < T_c$



(b) $T > T_c$

$$|\langle \mathcal{O}_2(t) \rangle| = C_1 \exp\left(-t/t_{\rm relax}\right) + C_2$$

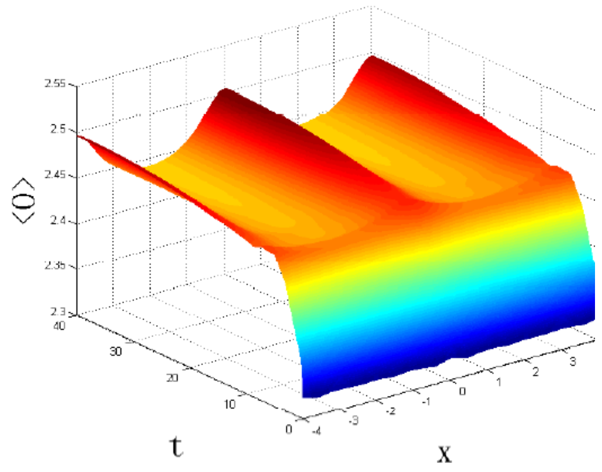
$$|\langle \mathcal{O}_2(t) \rangle| = C \exp\left(-t/t_{\rm relax}\right)$$

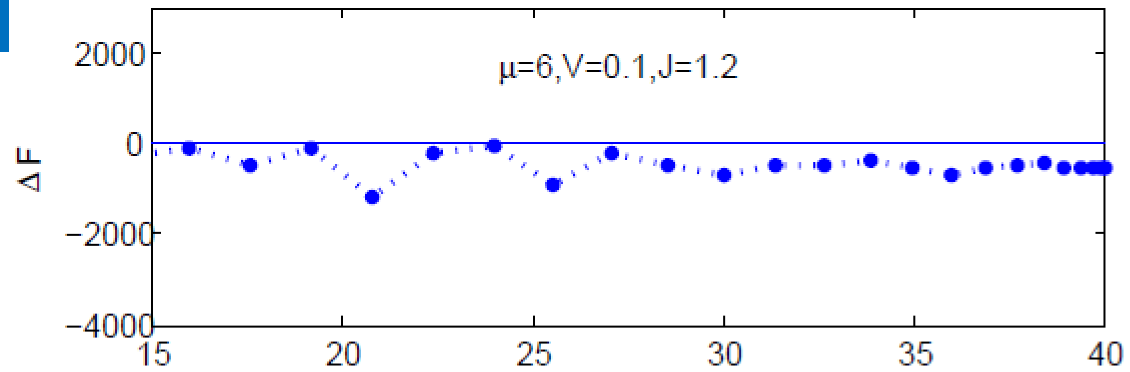
$$\tilde{\psi}(t=0,z) = \frac{\mathcal{A}}{\sqrt{2\pi}\,\delta} \exp\left[-\frac{(z-z_m)^2}{2\delta^2}\right]$$

$$\psi = z\psi_1(t) + z^2\tilde{\psi}(t,z)$$

Exponential growth

Oscillations in space

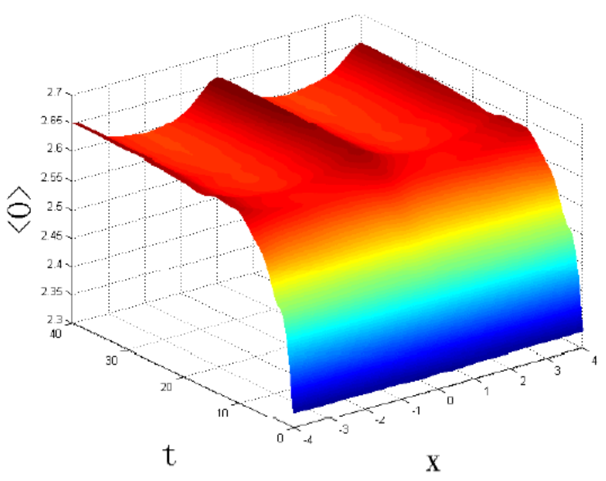




$$\Psi \approx z\psi_1 + \psi(x,t)z^2$$

$$\psi_1(t) = \mathrm{J}\tanh vt$$

$$\langle O \rangle \sim \psi(x,t)$$



Probe limit

AGG, Zhang, Bi,arXiv:1308.5398

Basu et al., arXiv:1308.4061

Conservation laws!

Oscillations in space: BdG

$$\hat{\xi} = -\vec{\nabla}^2/2m - \mu$$

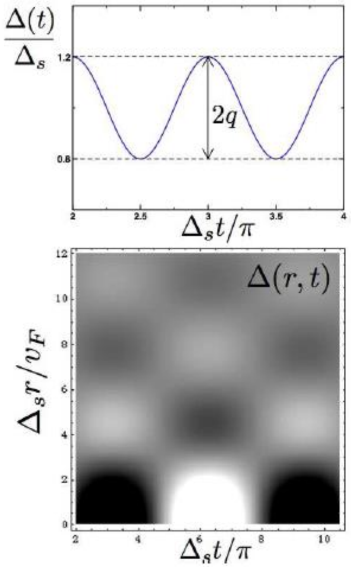
$$i\dot{u}_{\mathbf{p}}(\mathbf{r},t) = \hat{\xi}u_{\mathbf{p}}(\mathbf{r},t) + \Delta(\mathbf{r},t)v_{\mathbf{p}}(\mathbf{r},t),$$

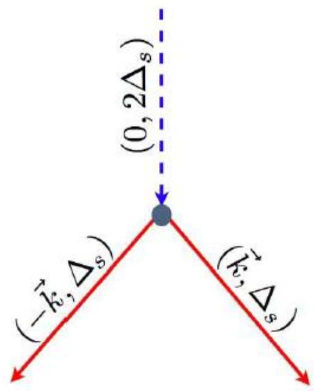
$$i\dot{v}_{\mathbf{p}}(\mathbf{r},t) = -\hat{\xi}v_{\mathbf{p}}(\mathbf{r},t) + \bar{\Delta}(\mathbf{r},t)u_{\mathbf{p}}(\mathbf{r},t)$$

$$\Delta(\mathbf{r},t) = \Delta(t) + \delta\Delta(\mathbf{r},t)$$

$$\delta\Delta(\vec{r},t) \approx \frac{Ce^{\nu_m t}\cos[\Delta_s(t-\tau)]}{\sqrt{\Delta_s t}}\frac{\sin(k_m R)e^{-R^2/l^2(t)}}{k_m R}$$

$$l(t) \approx \xi\sqrt{\Delta_s t} \qquad \nu_m \approx 2q\Delta_s$$

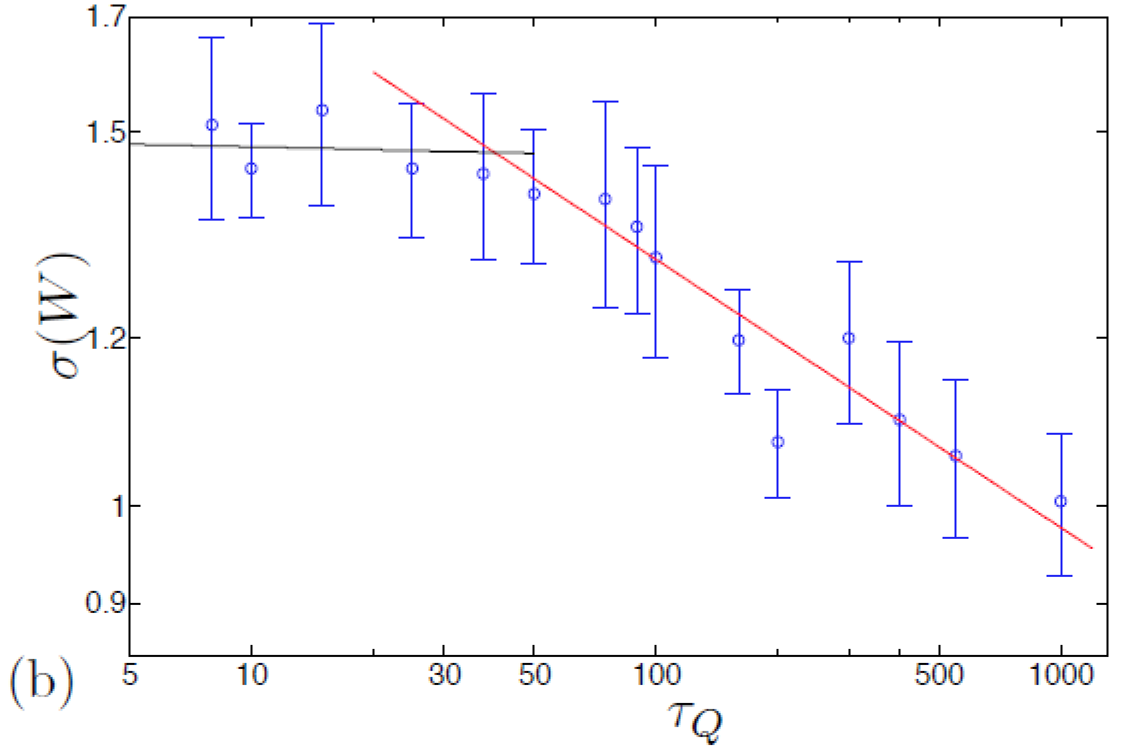




Conservation laws

Instability to spatial inhomogeneity

Dzero et al., EPL 85 (2009) 20004



Sonner, Campo, and Zurek arXiv:1406.2329

Defects in 1d holographic superconductor

Only Check of KZ scaling

Dual gravity theory

$$S_{\text{grav}} = \frac{1}{16\pi G_{\text{Newton}}} \int d^4x \sqrt{-G} \left[R + \Lambda + \frac{1}{e^2} \left(-\frac{1}{4} F_{MN} F^{MN} - |D\Phi|^2 - m^2 |\Phi|^2 \right) \right]$$

$$\Lambda = -3 \qquad m^2 = -2$$

Herzog, Horowitz, Hartnoll, Gubser

AdS_4

Eddington-Finkelstain coordinates

$$ds^2 = r^2 g_{\mu\nu}(t, \boldsymbol{x}, r) dx^\mu dx^\nu + 2 dr dt$$

Probe limit

$$0 = \nabla_M F^{NM} - J^M,$$

$$0 = (-D^2 + m^2)\Phi.$$

EOM's:

PDE's in x,y,r,t

$$= 0$$

Boundary conditions:

$r \to \infty$

$$\Psi = \frac{\psi^{(1)}(x,y,t)}{r} + \frac{\psi^{(2)}(x,y,t)}{r^2} + ..$$

$$A_t = \mu - \rho/r$$

hep-th/9905104v2
arXiv:1309.1439
Science 2013

Drive:

$$\epsilon(t) = t/\tau_Q \qquad t_i = (1 - T_i/T_c)\tau_Q$$

$$t \in (t_i, t_f) \qquad t_f = \left(1 - T_f/T_c\right)\tau_Q$$

No solution of Einstein equations but do not worry, Hubeny 2008

Dictionary:

$$\langle O_2 \rangle \sim \psi_2$$

Stochastic driving

$$\psi^{(1)} = \varphi(t, x)$$

$$\langle \varphi^*(t, x)\varphi(t', x')\rangle = \zeta\delta(t - t')\delta(x - x')$$

Field theory: $\zeta(T, \nu)$

Quantum/thermal fluctuations

Gravity: $\zeta \propto 1/N^2$

Hawking radiation

Predictions:

Slow quenches:

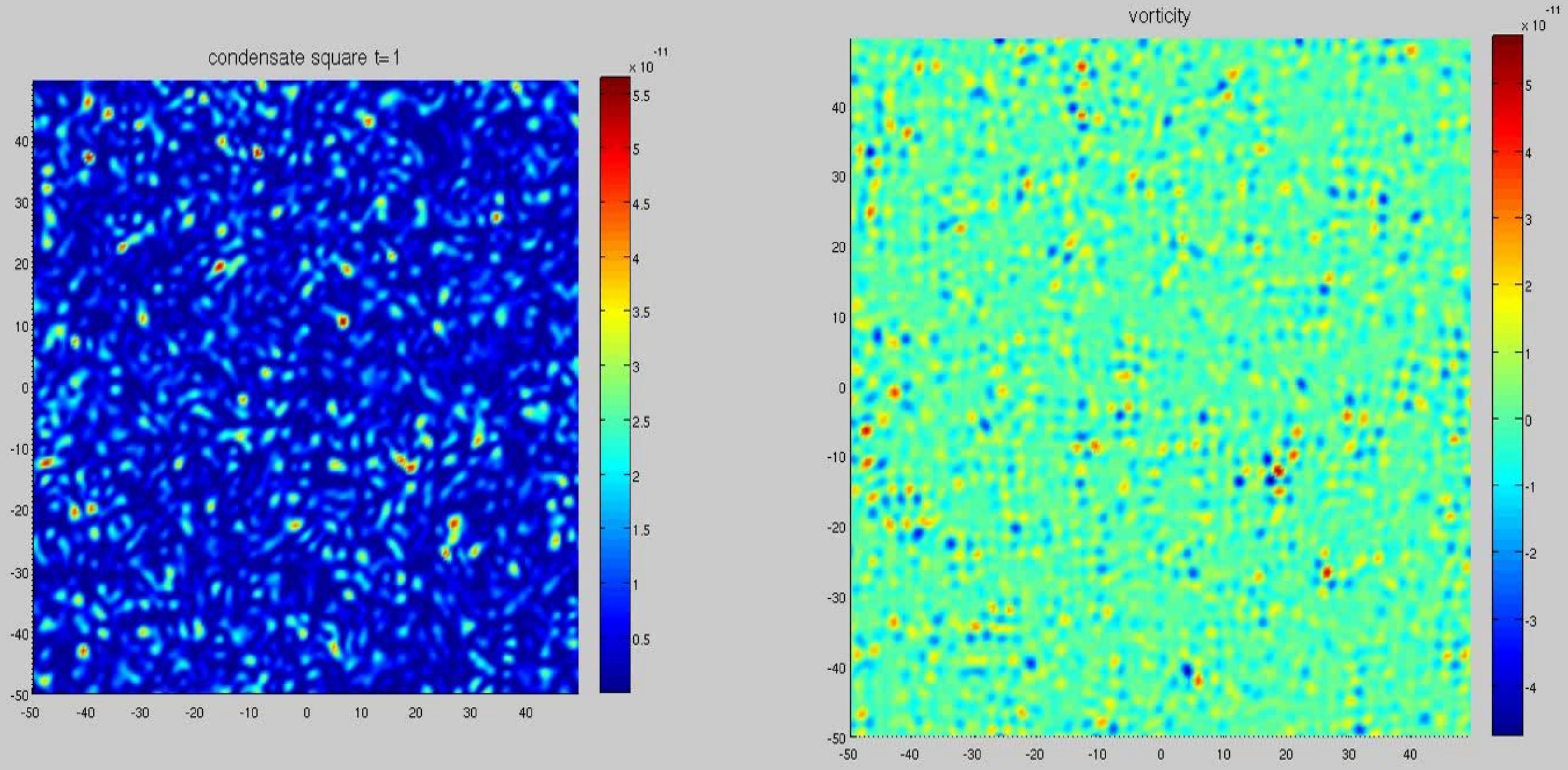
$$C(t,r) \sim |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}, \quad |\psi|^2(t) \sim \tilde{\varepsilon} t_{\rm freeze} \bar{t} e^{a_2 \bar{t}^2}, \quad \ell_{\rm co}(t) \sim \xi_{\rm freeze} \sqrt{\bar{t}}$$

$$\frac{t_{\rm eq}}{t_{\rm freeze}} \sim \sqrt{\log \frac{N^2}{\sqrt{\tau_Q}}} \qquad \rho \sim \frac{1}{\sqrt{\log \frac{N^2}{\sqrt{\tau_Q}}}} \rho_{\rm KZ}$$

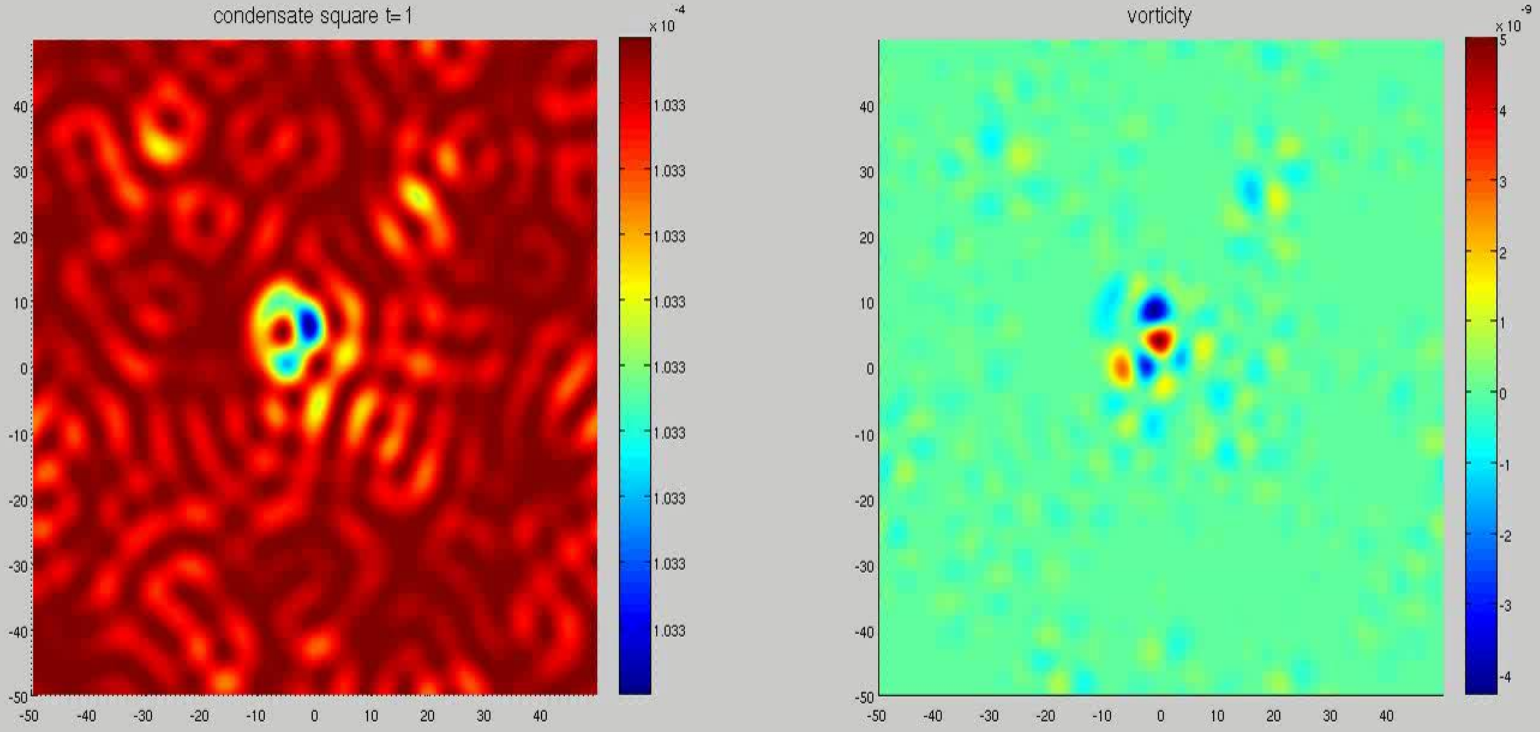
Fast quenches:

$$C(t,r) = |\psi|^2(t) e^{-\frac{r^2}{2\ell_{\rm co}^2(t)}}, \qquad |\psi|^2(t) \sim \zeta \exp\left[2b(t - t_{\rm freeze})\epsilon_f\right]$$

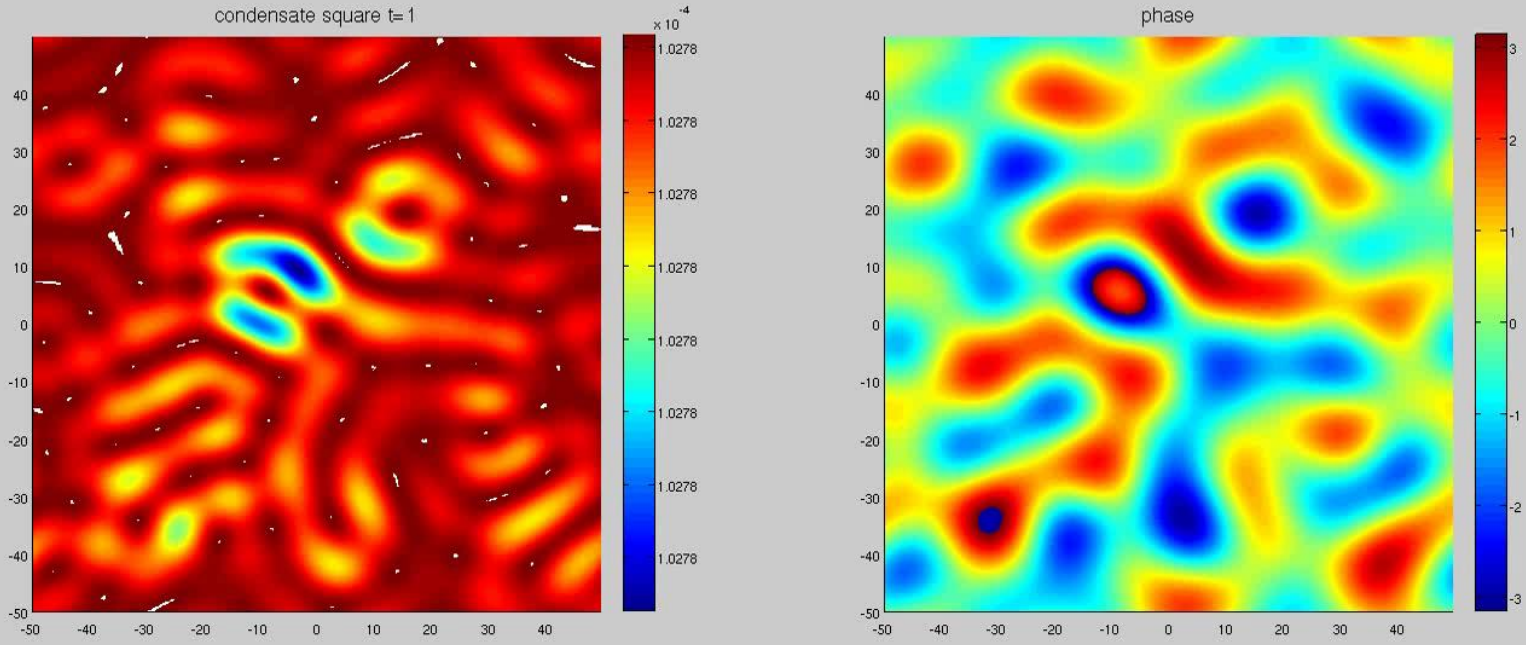
$$\ell_{\rm co}^2(t) = 4a(t - t_{\rm freeze}) \qquad \rho \sim \frac{\epsilon_f}{\log \frac{N^2}{\epsilon_f}}$$



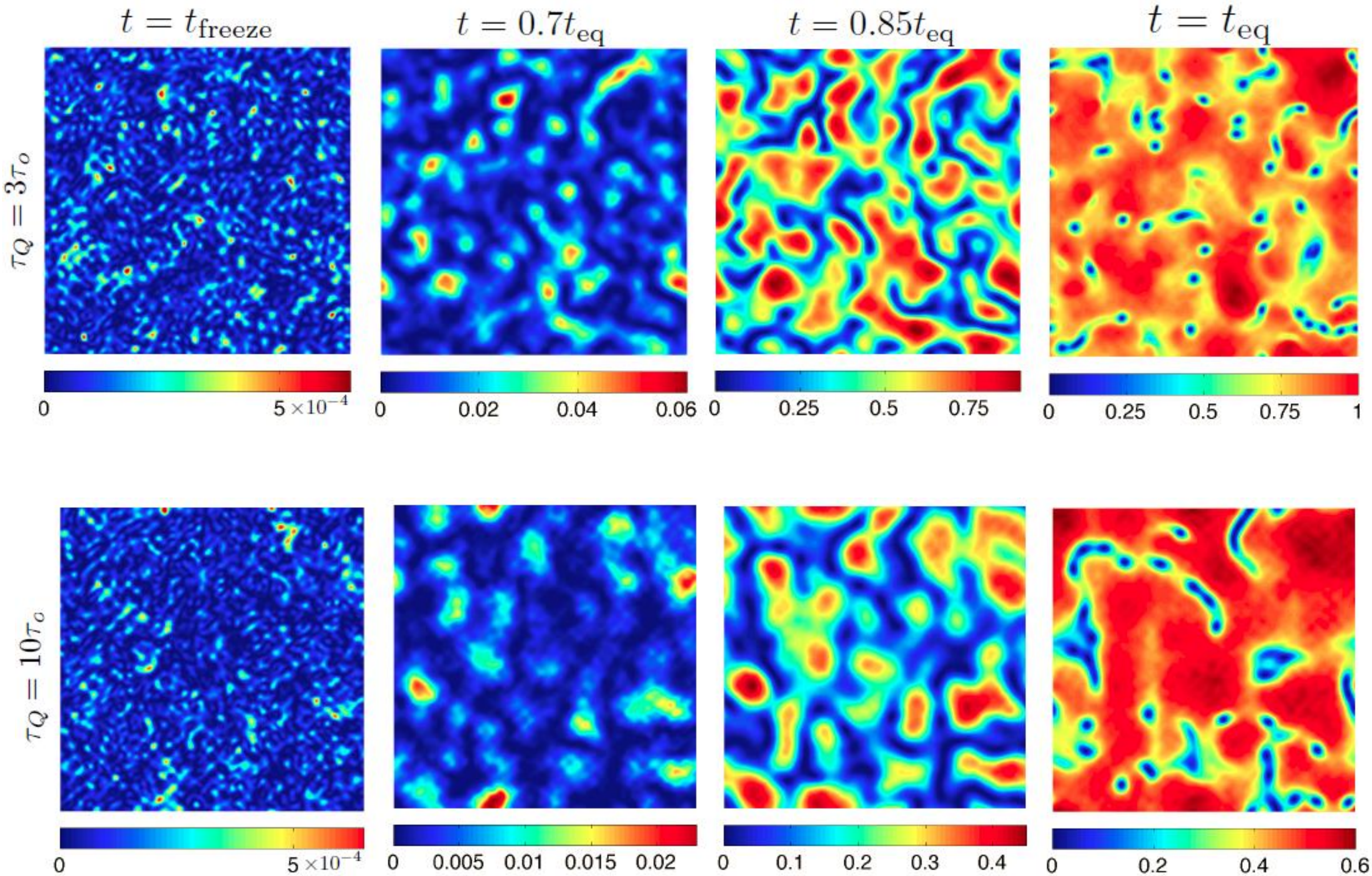
Slow quench



Fast Quench

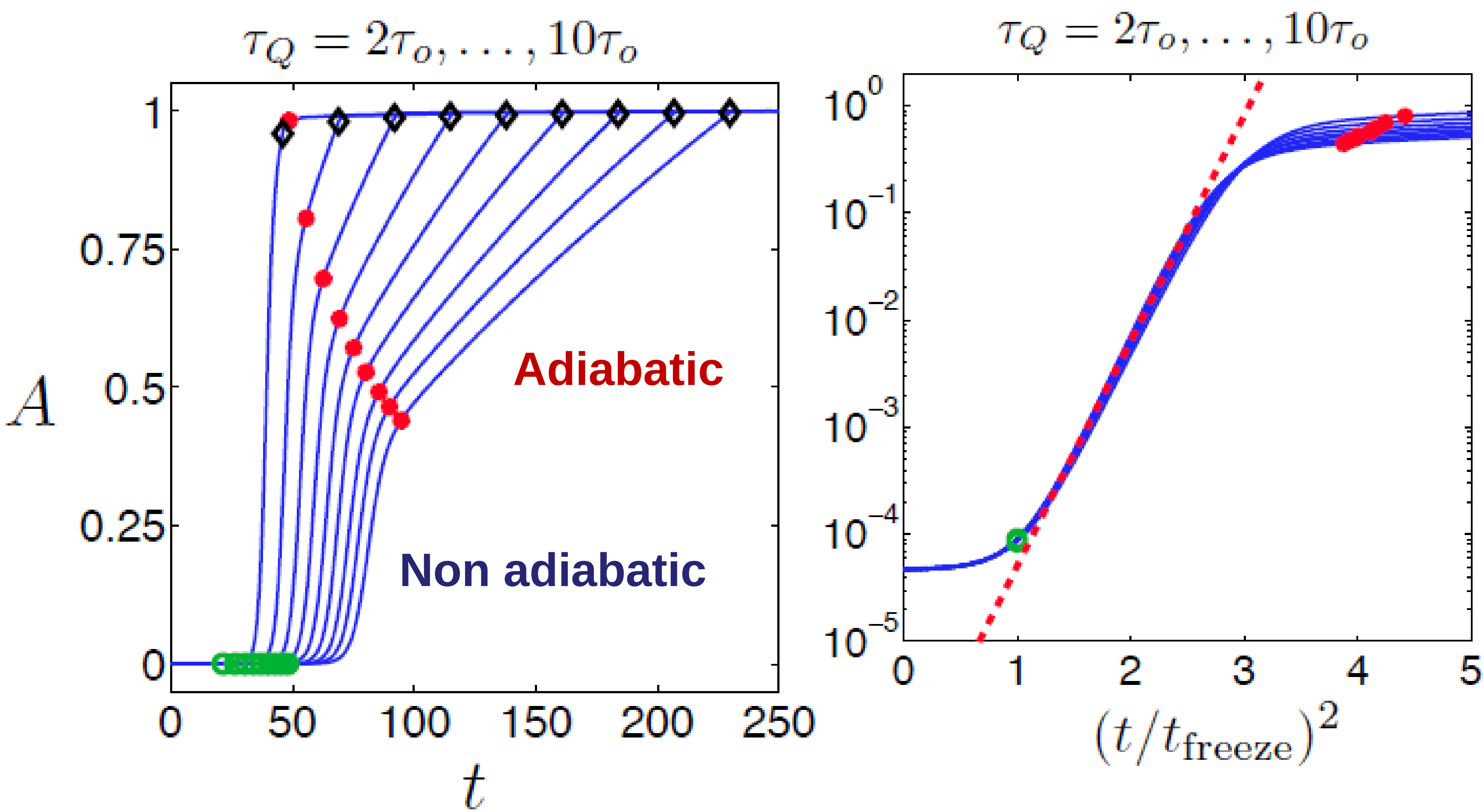


Slow Condensate-Phase

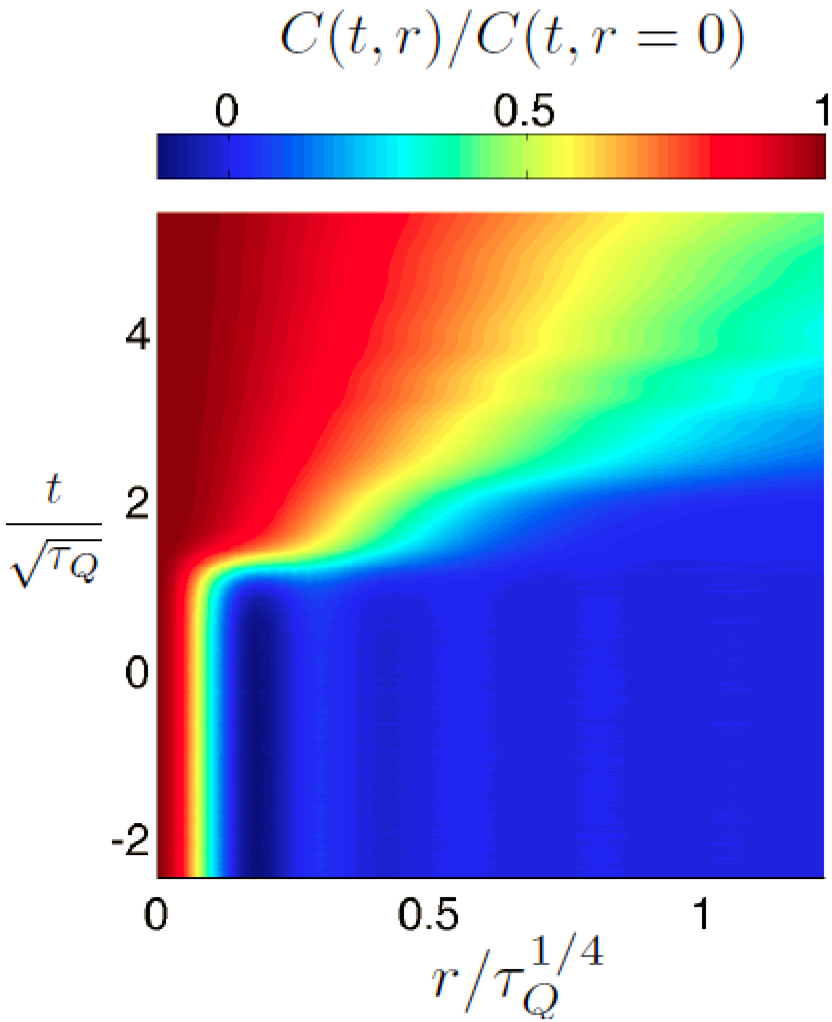


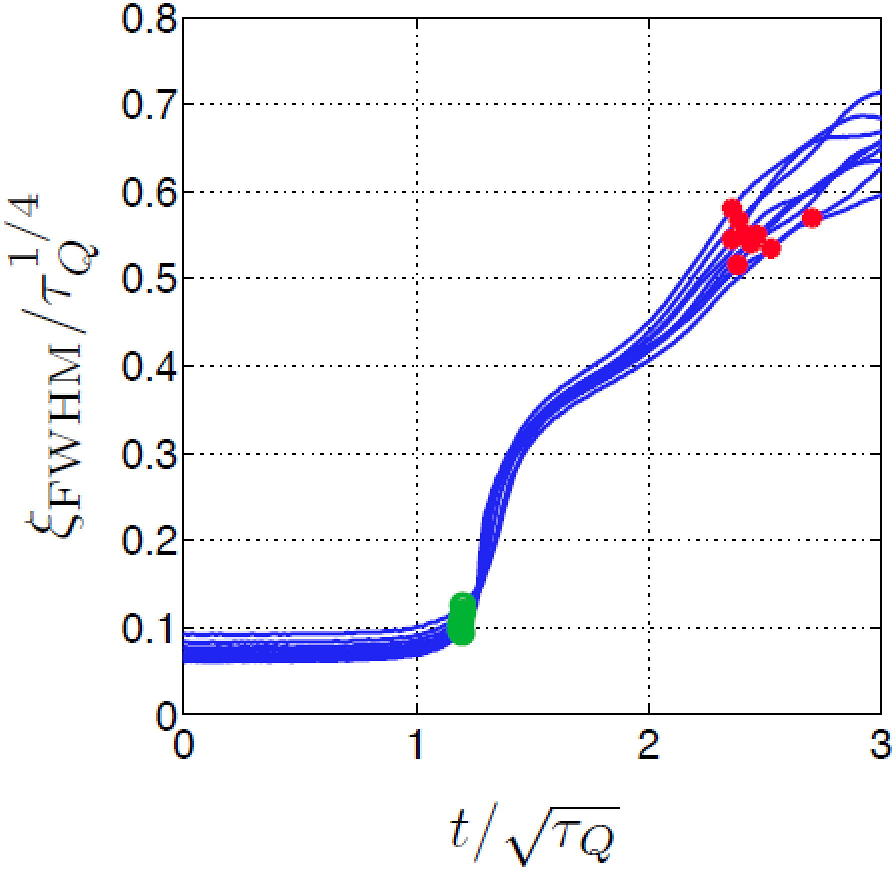
t_{eq} *is the relevant scale*

$$A(t) = \frac{1}{M}\sum_{i=1}^{M}\frac{a_i(t)}{a_i(\infty)}, \qquad a_i(t) \equiv \int d^2x\, |\psi_i(t,\boldsymbol{x})|^2$$



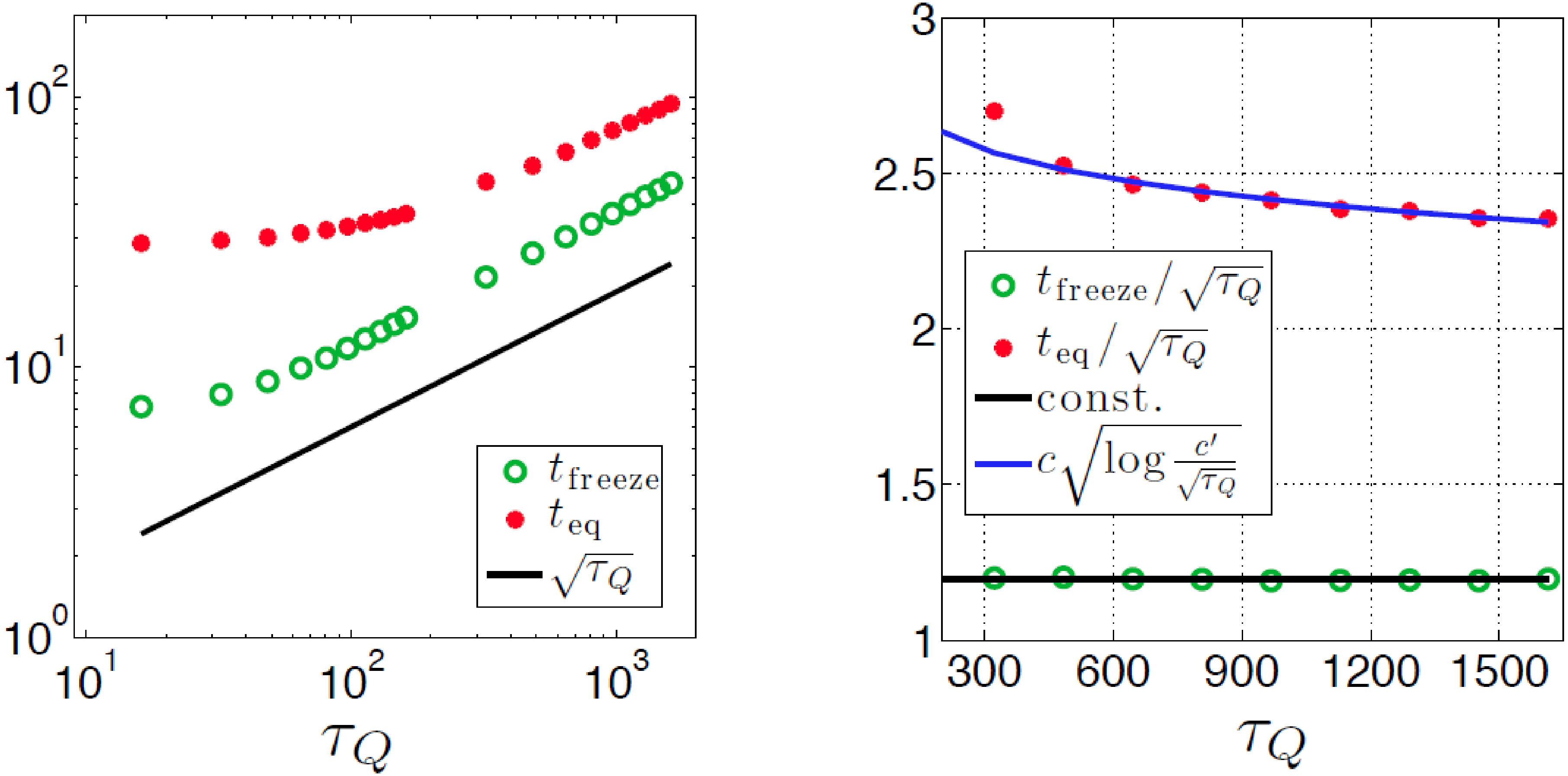
$$|\psi|^2(t) \sim \tilde{\varepsilon} t_{\mathrm{freeze}} \bar{t} e^{a_2 \bar{t}^2}$$





Strong coarsening

$t > t_{freeze}$



$$t_{freeze} \ll \ t_{eq}$$

Correct scaling

Slow

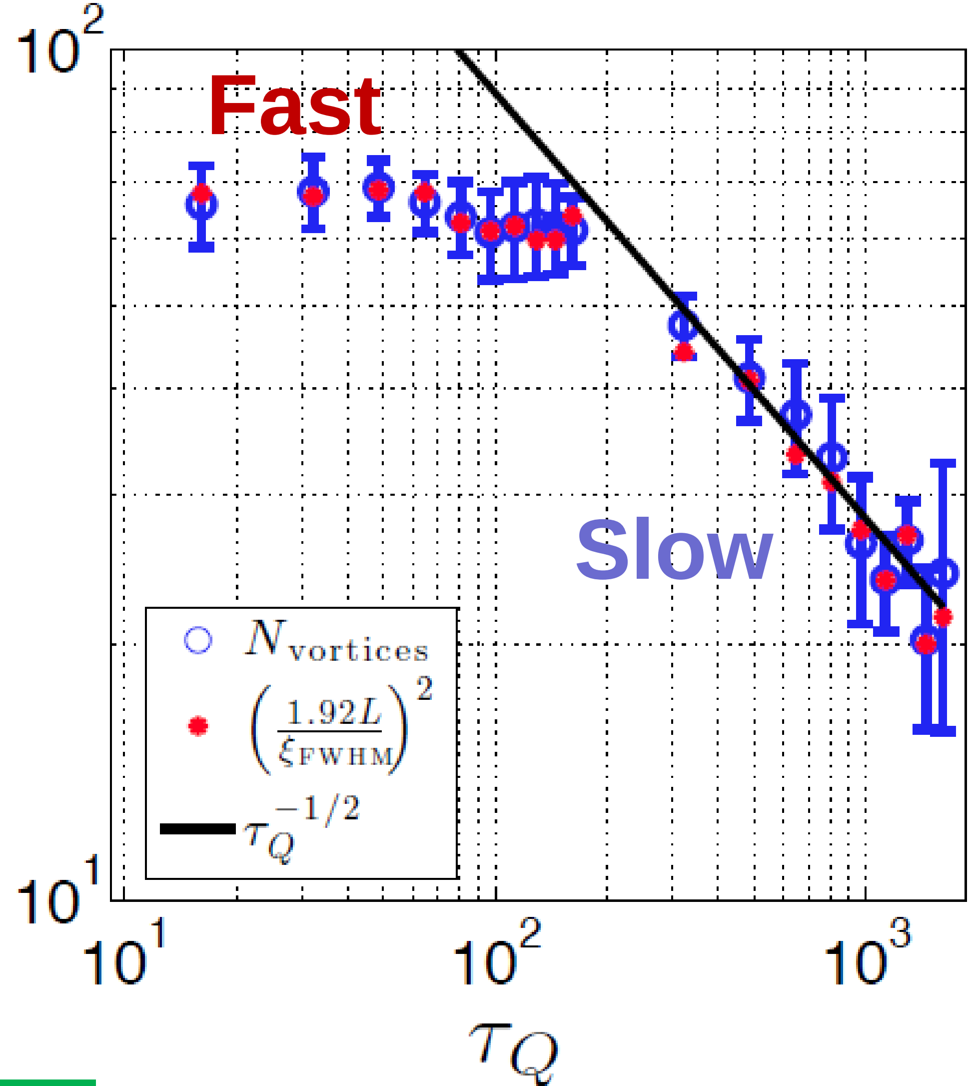
$$\rho \sim \frac{\rho_{KZ}}{\left(\log N^2/\tau_Q{}^{1/2}\right)^{1/2}}$$

Fast

$$\rho \sim \frac{\epsilon_f}{\log(\frac{N^2}{\epsilon_f})}$$

Relevant for ^{4}He ?

$$t_{\rm eq} \sim [\log \tau_Q]^{1/(1+\nu z)} t_{\rm freeze}$$

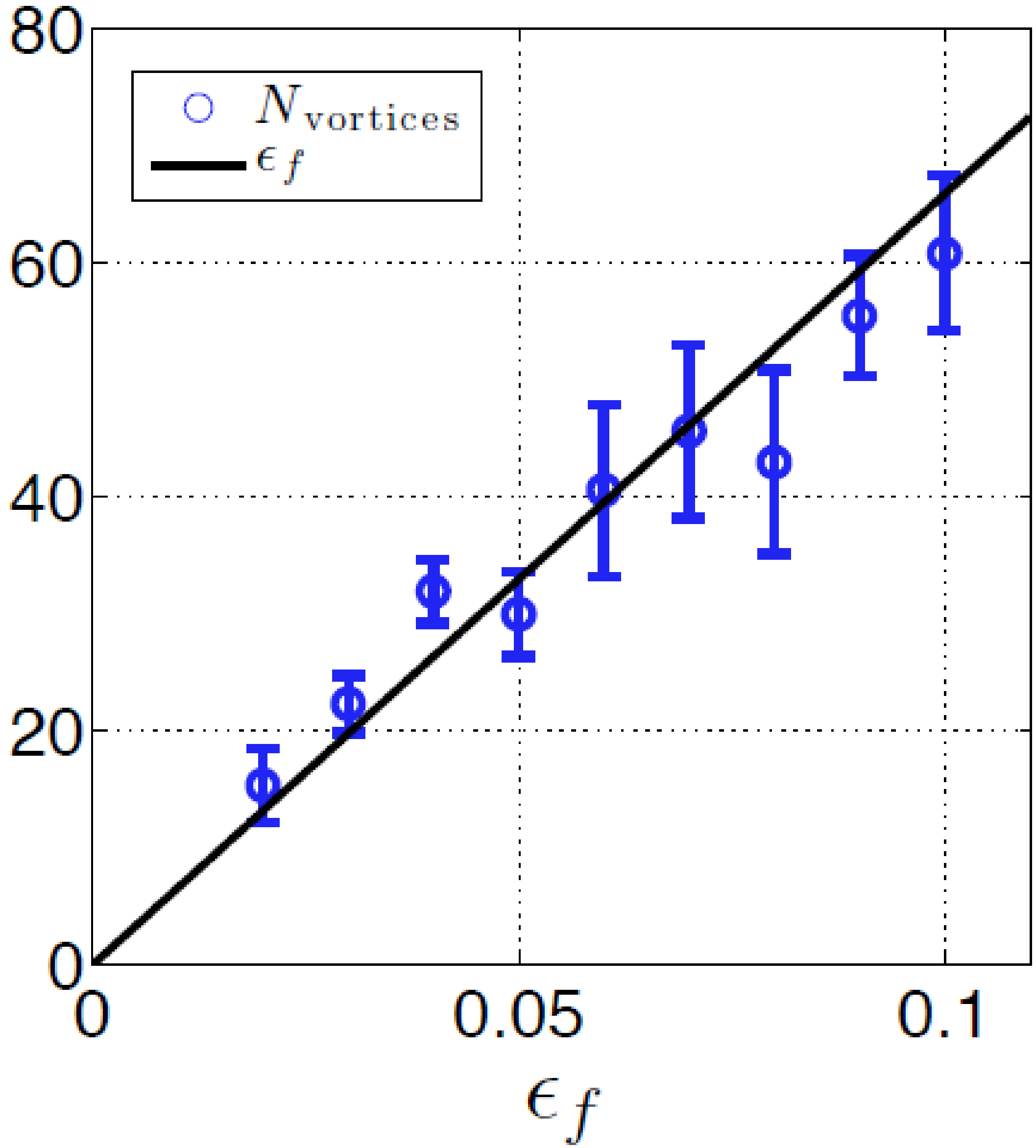


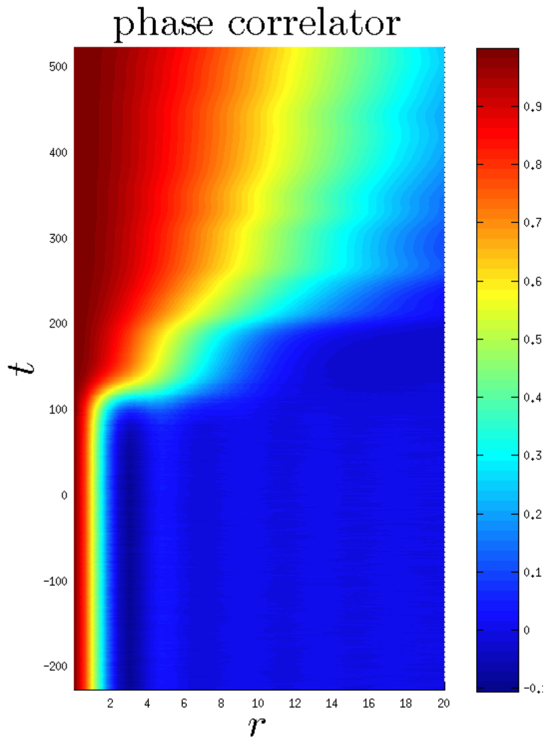
~25 times less defects than KZ prediction!!

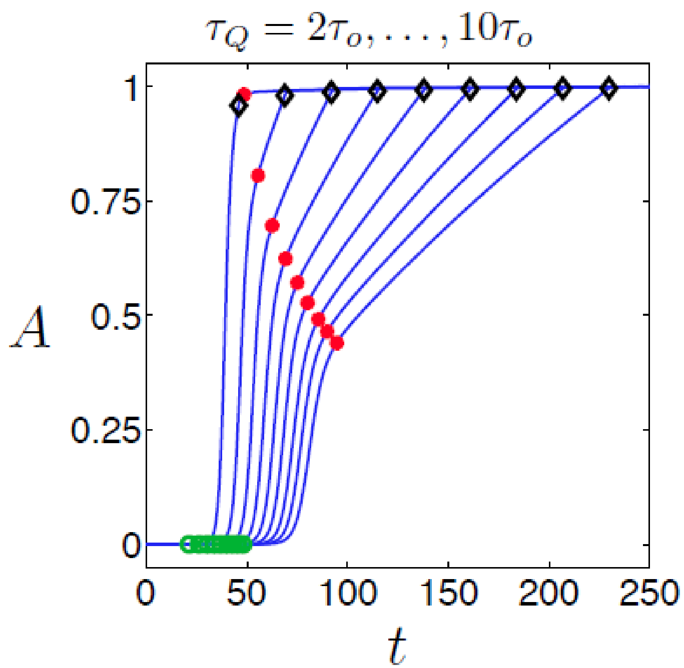
Fast quenches

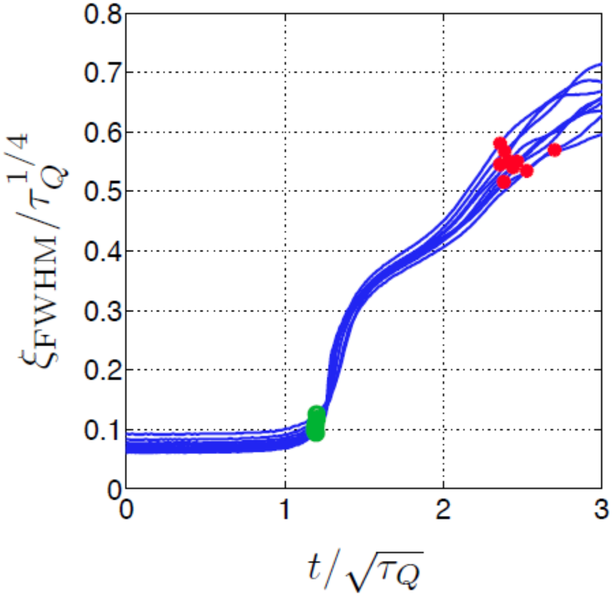
$$\rho \sim \frac{\epsilon_f}{\log(\frac{N^2}{\epsilon_f})}$$

T>T_c dynamic irrelevant









time

Freezing

Condensate formation

Defect generation

Phase coherence ?

H O L O G R A P H Y

Physics beyond Kibble-Zurek

More efficient than SGPE

Cracking thermalization?

^4He?

Vortex physics

BKT transition

ευχαριστίες