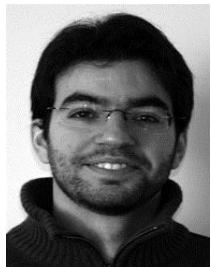


# Enhancing Tc in nano-structured materials

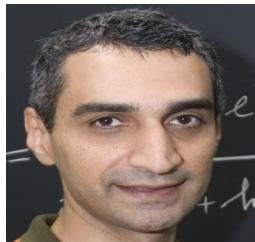
Mayoh, AGG, arXiv:1311.0295

**Antonio M. García-García**

Cavendish Laboratory Cambridge University, Lisbon University



Pedro Ribeiro  
Lisbon



Yuzbashyan  
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Urbina  
Regensburg



Richter  
Regensburg



Bermudez  
Cambridge



Way  
Cambridge



Sangita Bose  
Bombay



Altshuler  
Columbia



Klaus Kern  
Stuttgart

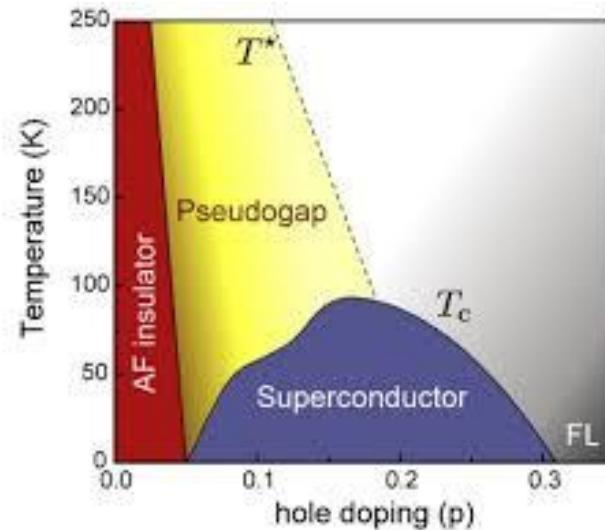


Mayoh  
Cambridge

# Superconductivity



## Mavericks



Quantum critical points ©

Cuprates

~100K

1986

Mueller & Bednorz

$MgB_2$

39K

2001

Akimitsu

FeSC

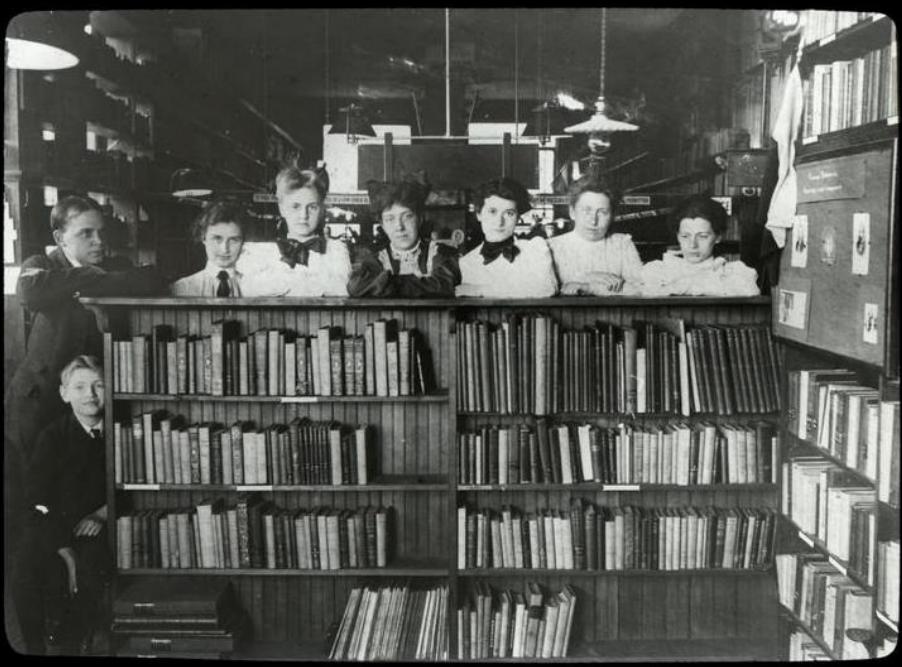
~50K

2006

Hotsono

Pb ~7K Al ~1K Sn ~3.7K Nb ~9.3K

## Librarians



BCS + ....

Thin films  
Josephson Junctions  
Nanowires

Thinner

Cleaner

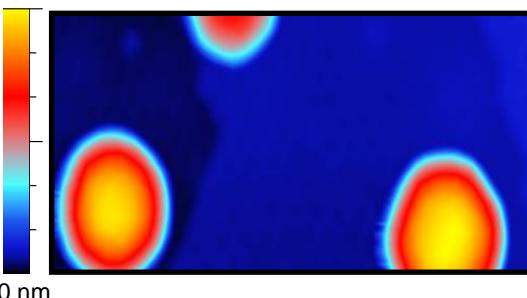
Smaller

Granular

Abeles, Tinkham, Devoret, Goldman, Xue, Kern, Di Fazio, Schoen, Halperin, Leggett, Blatt....

Control

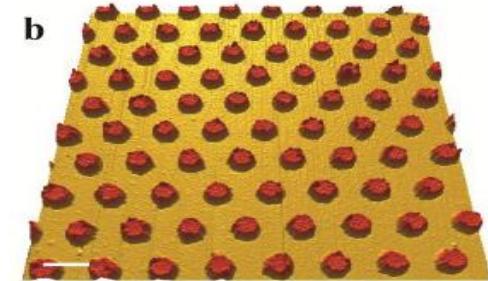
7 nm



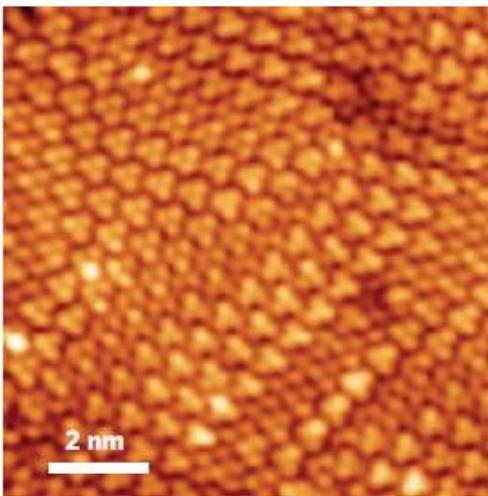
# Grains

## Far from equilibrium

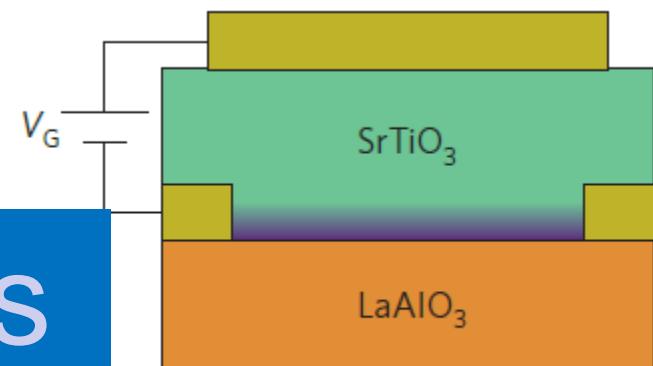
### Higher $T_c$ ?



## Arrays



## Interfaces

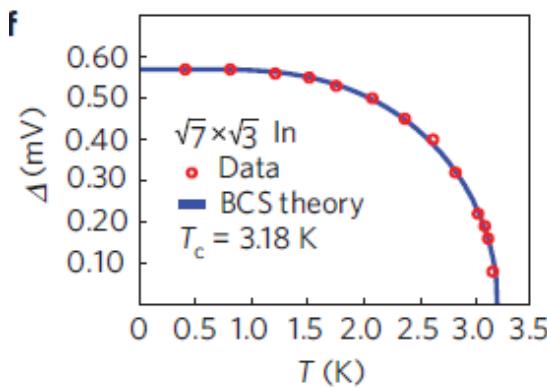
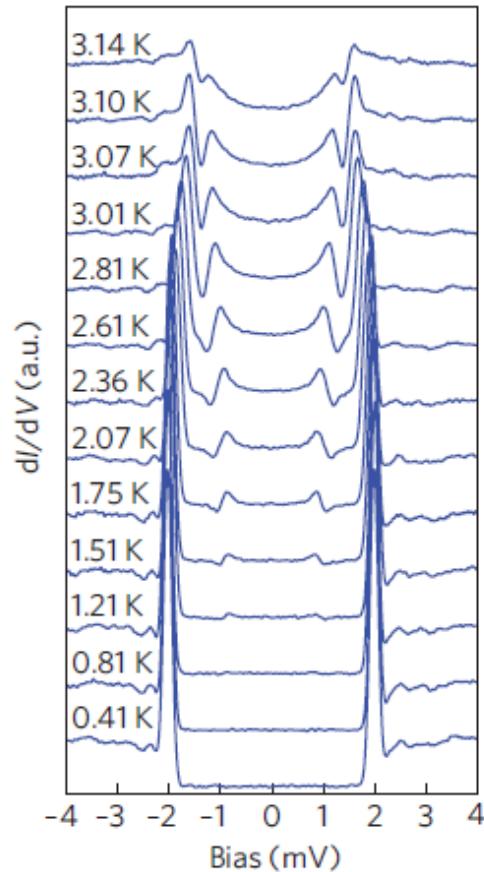
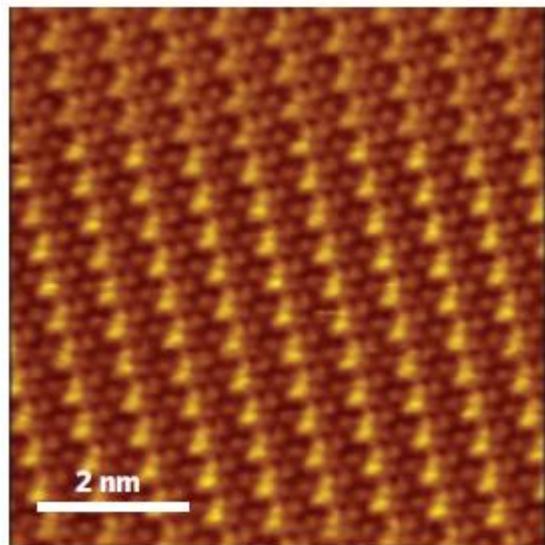


# Superconductivity in one-atomic-layer metal films grown on Si(111)

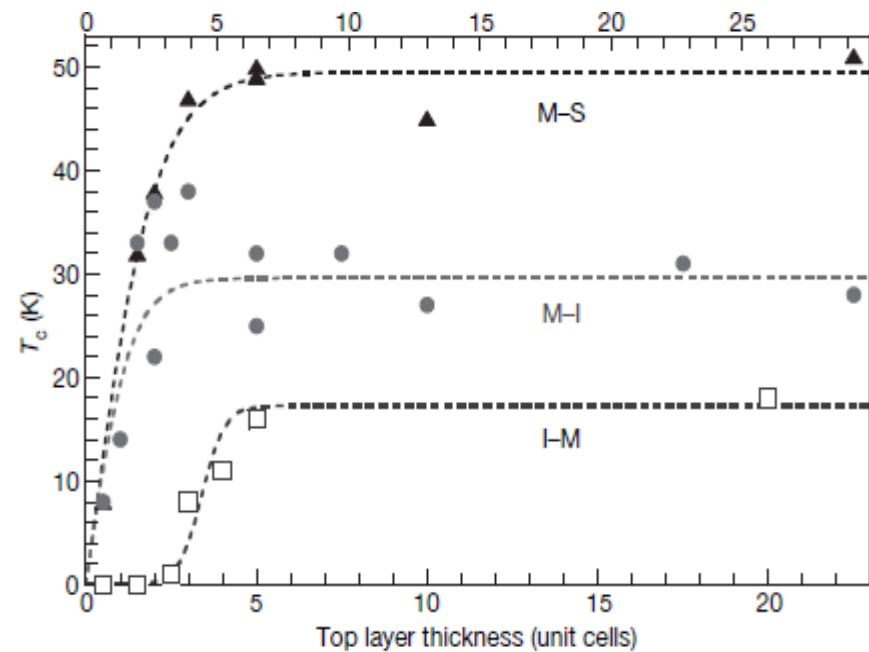
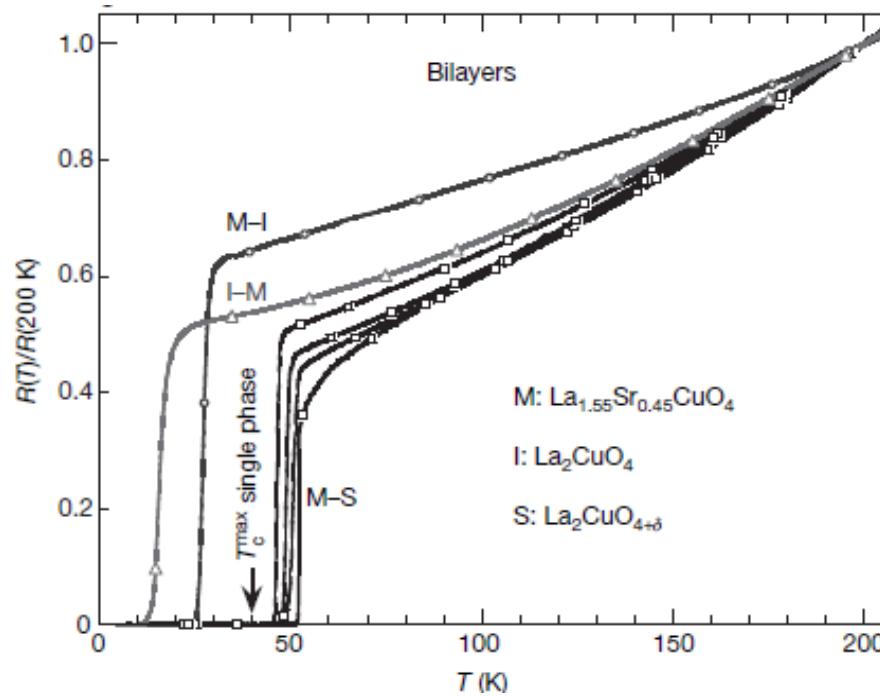
Epitaxial  
growth

STM

No impurities



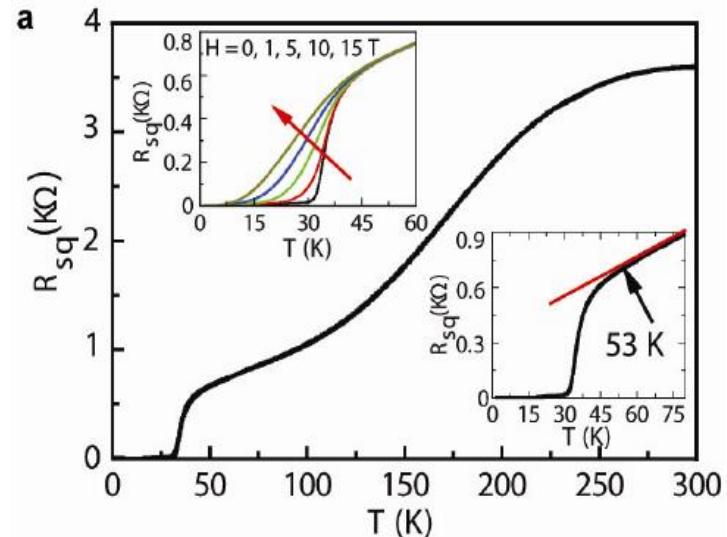
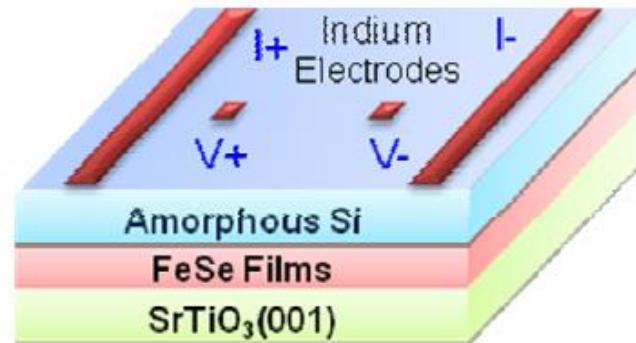
# Cuprates high $T_c$ Heterostructures



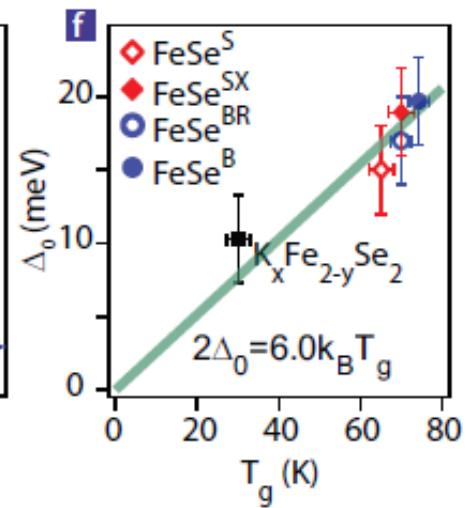
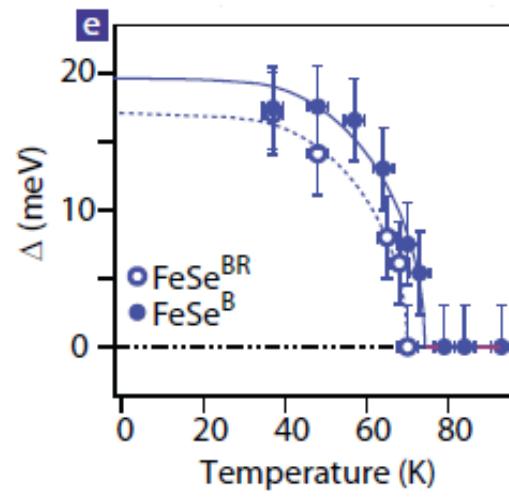
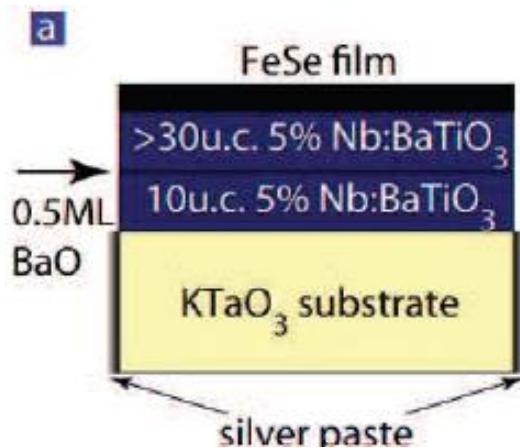
Bozovic et al., Nature 455, 782 (2008)

Higher  $T_c$ !!

# Iron Based Heterostructures



Xue et al., Nature Communications 3, 931 (2013)



# Enhancement

# Single Grains

# BCS superconductivity

$$\frac{2}{g} = \int_{-E_D}^{E_D} \frac{\nu(\varepsilon)}{\sqrt{\Delta^2 + \varepsilon^2}} d\varepsilon$$

$$\nu(\varepsilon) = \sum_i c_i \delta(\varepsilon - \varepsilon_i)$$

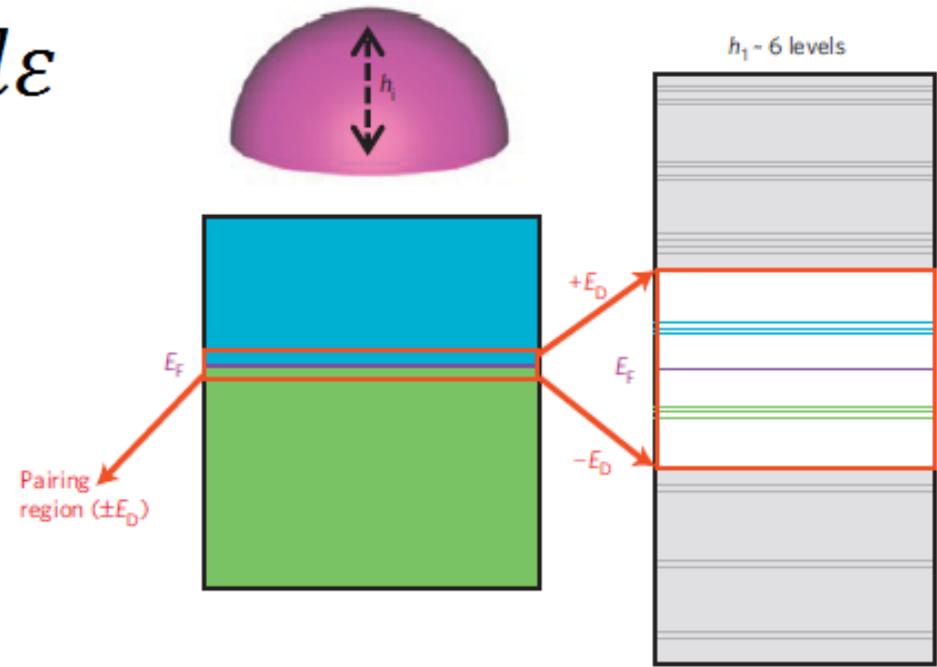
$V \rightarrow \infty$   
 $\Delta \sim \varepsilon_D e^{-1/\lambda}$

$V$  finite  
 $\Delta = ?$

## Shell Effects

Parmenter, Phys. Rev. 166,  
392 (1967)

# Finite size effects



## Level Degeneracy

$L \sim 5\text{nm}$



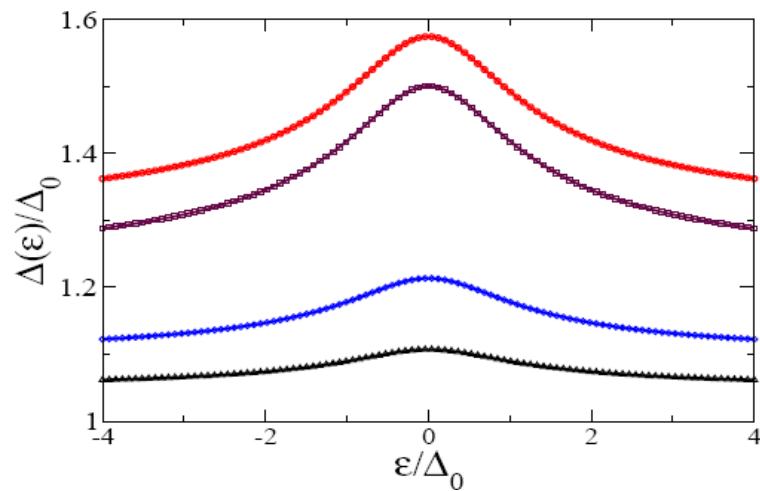
$20T_c!$

# 3d chaotic

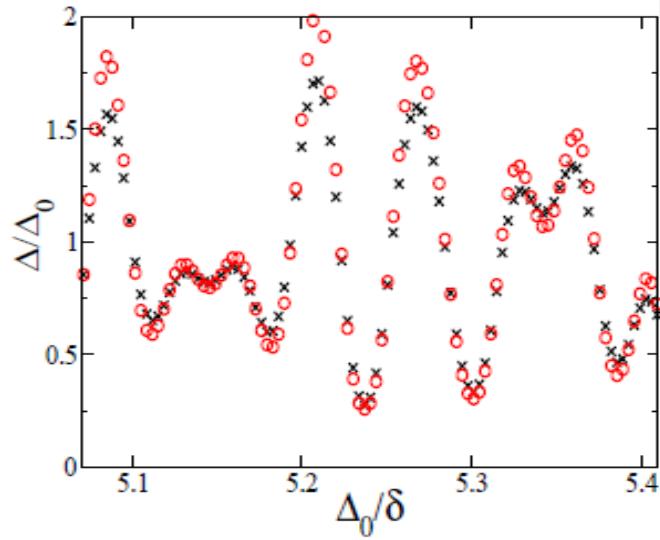
$$\Delta \gg \delta$$

$$\frac{1}{k_F L} \ll 1$$

AGG, Altshuler, et al., PRL 100, 187001 (2008)  
PRB 83, 014510 (2011)



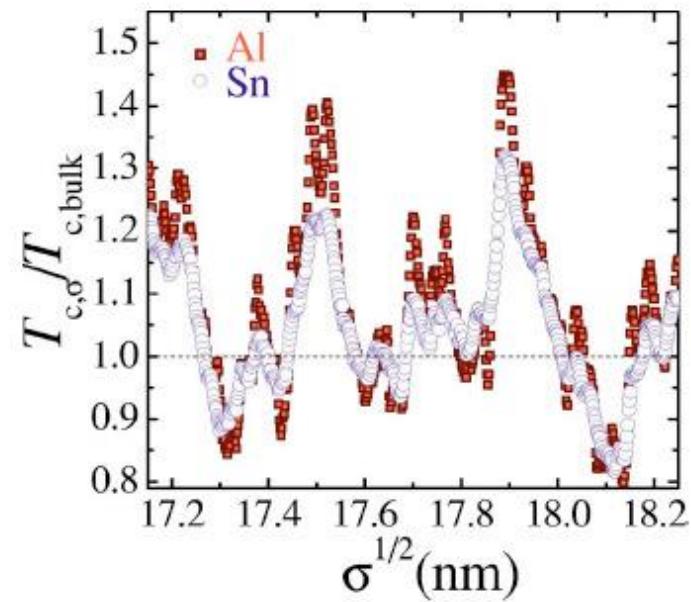
# 3d integrable



Kresin, Ovchinnikov,  
Boyaci (2007) Spheres

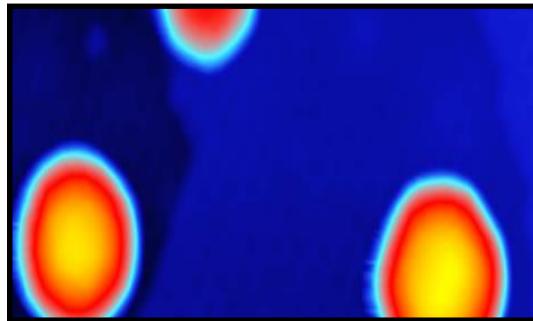
Heiselberg (2002):  
harmonic potentials.

Devreese (2006): Richardson  
equations in a box



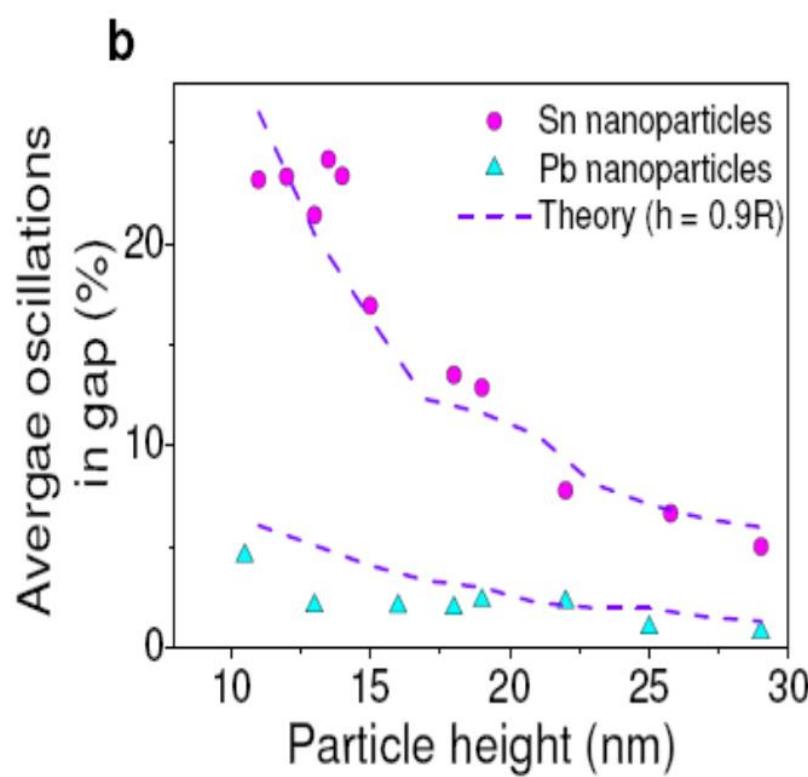
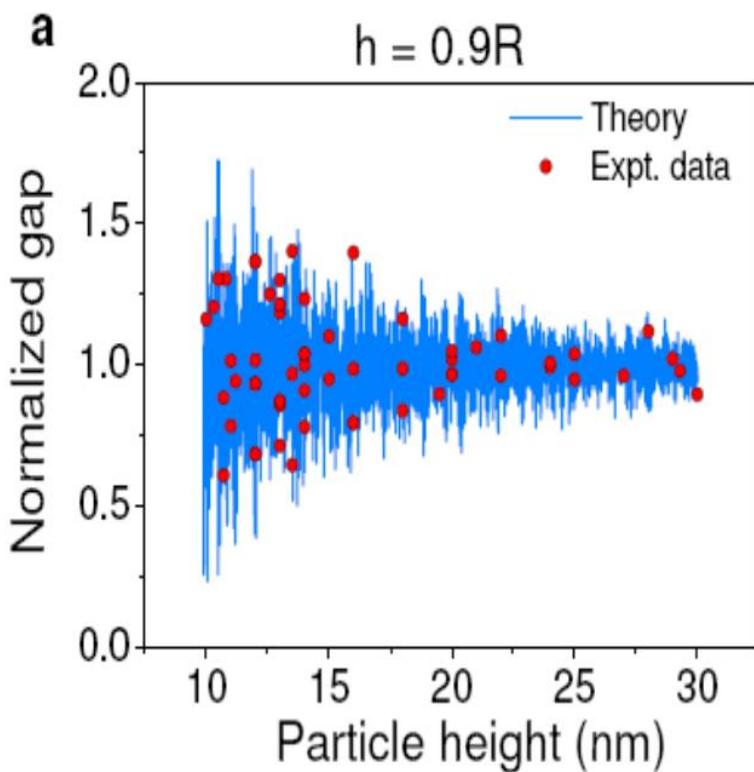
Peeters, et al, (2005-): BCS,  
BdG in a wire, cylinder

7 nm

 $R \sim 4\text{-}30\text{nm}$ , Pb, Sn

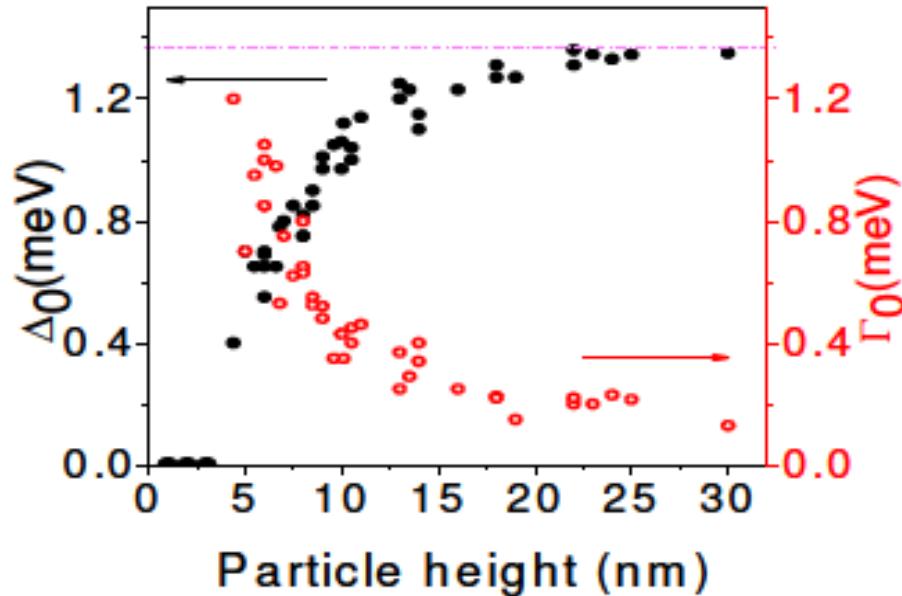
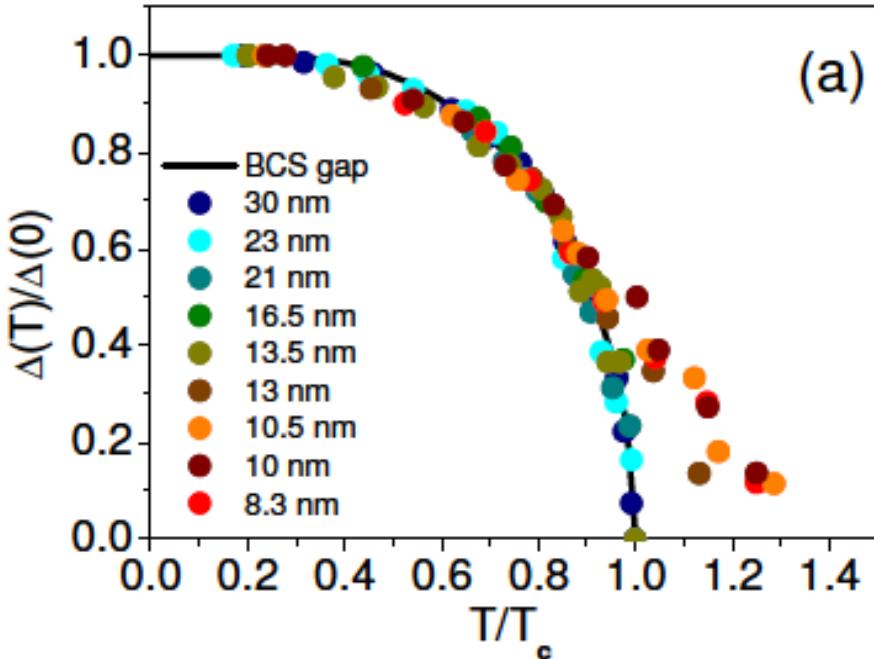
A gap is still observed

Almost hemispherical



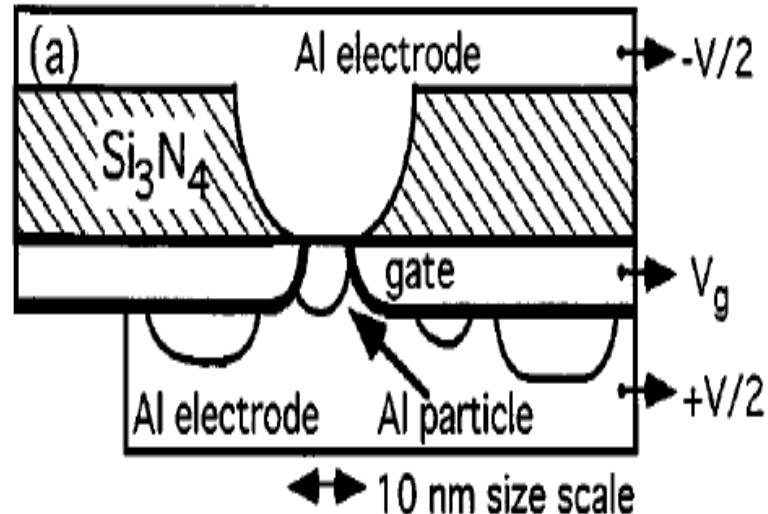
# Beyond mean field

$\Delta \sim \delta$



Superconductivity?

1959

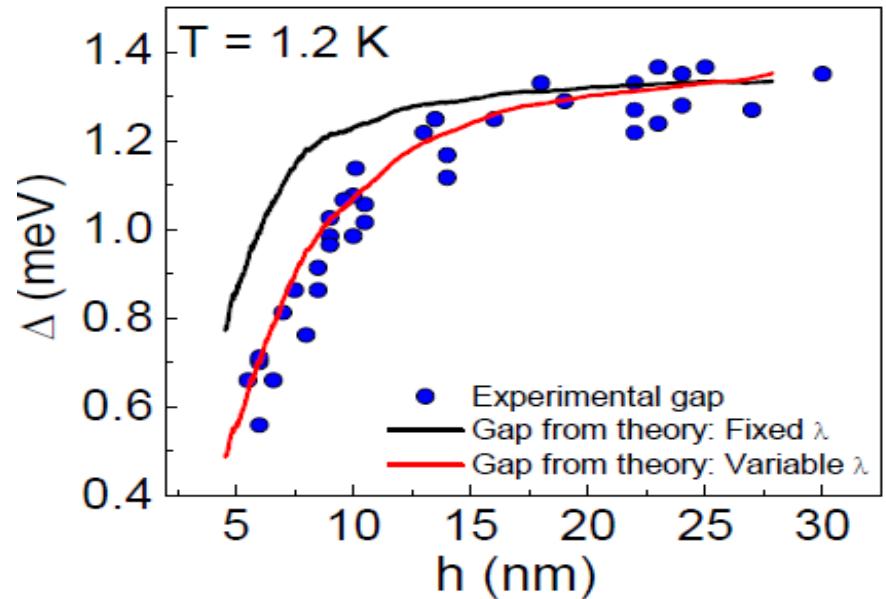
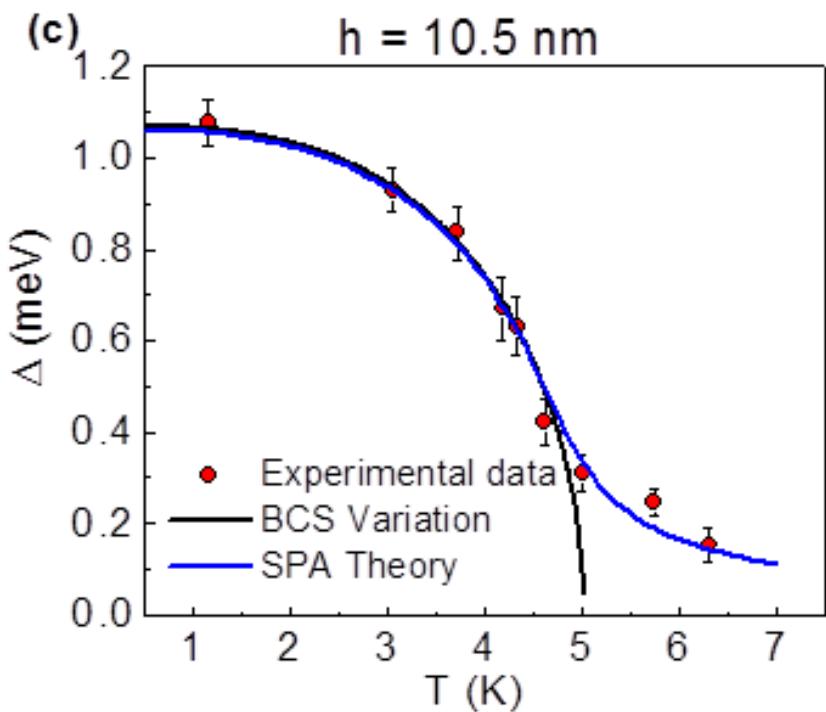


Tinkham 95

# Quantum Fluctuations

## Richardson's equations

and



# Thermal Fluctuations

## Static Path Approach

Brihuega, AGG, Ribeiro, Bose, Kern  
PRB 84,104525 (2011)  
Editor's Suggestion

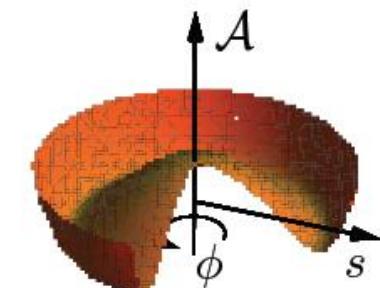
## Divergences at intermediate $T$

Rossignoli and Canosa  
Ann. of Phys. 275, 1, (1999)

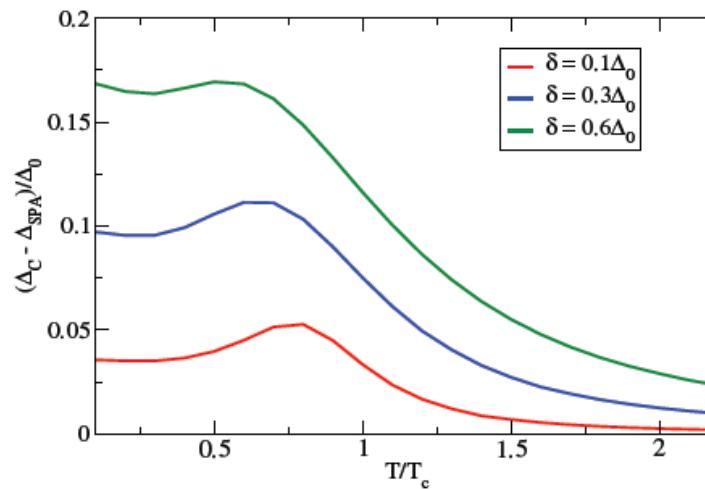
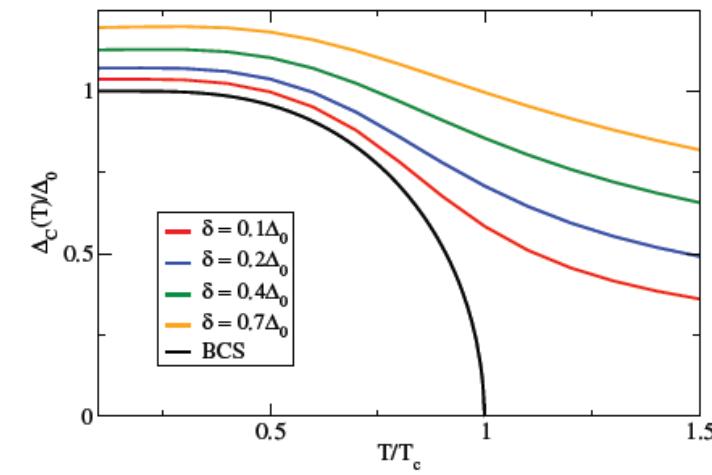
Harmful Zero Modes

Polar coordinates

Quantum fluctuations  $\sim$  Charging effects



$$\Delta(\tau) = s(\tau)e^{i\phi(\tau)}$$



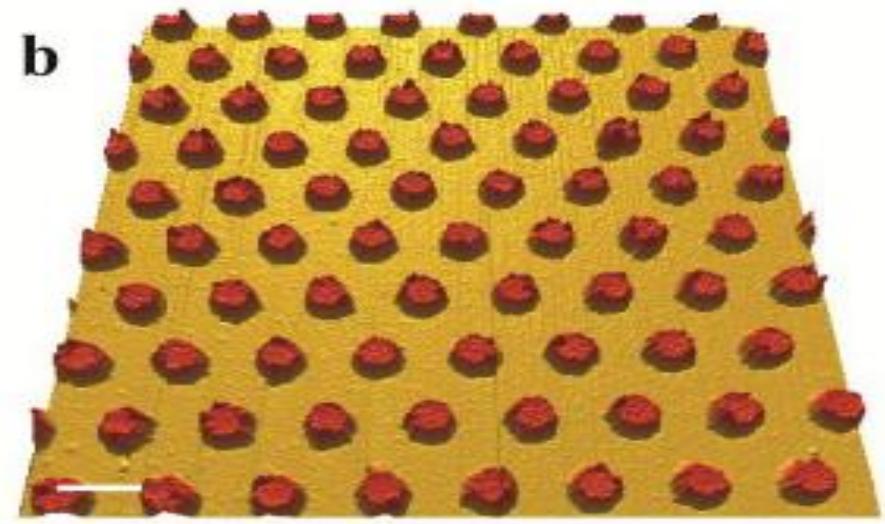
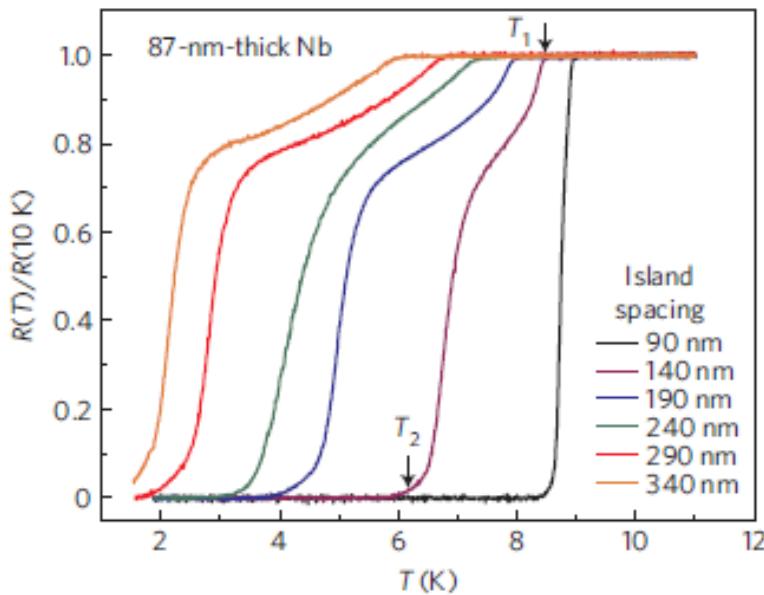
# True phase coherence in single nanograins?

## Josephson array?

# No

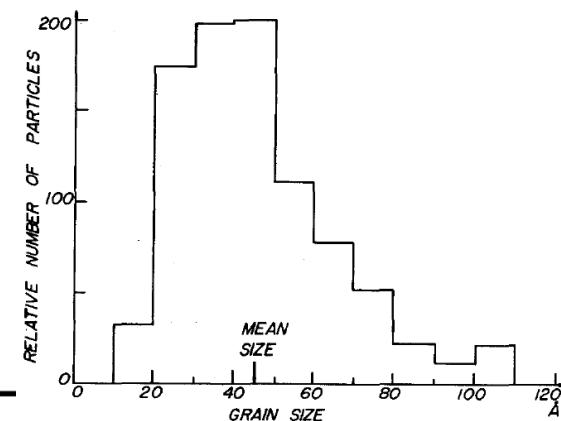
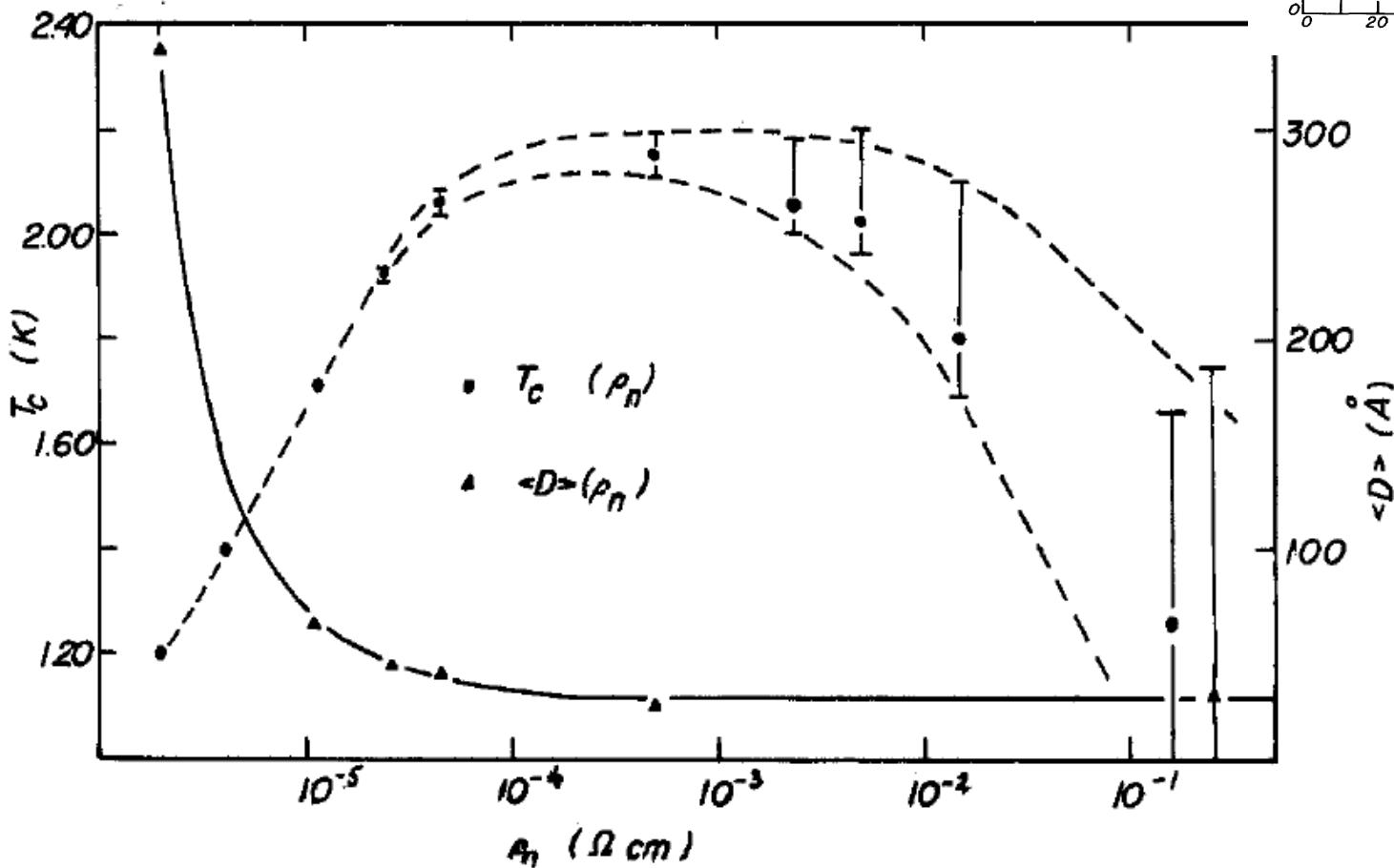
$$\Delta N \Delta \phi \geq \hbar$$

# Maybe



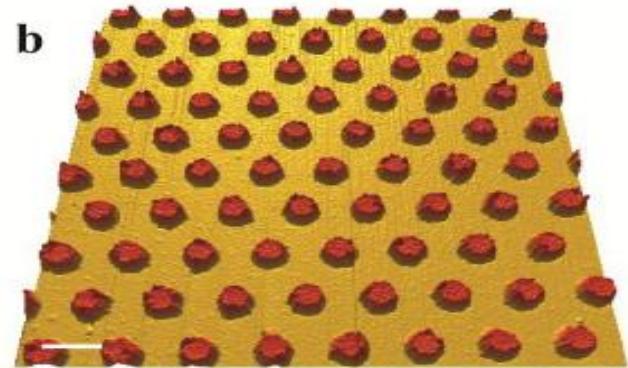
Mason, et al, Nature Physics 8 59 (2012)

# Al evaporated on a glass substrate



# Engineering granular materials

Optimal but realistic



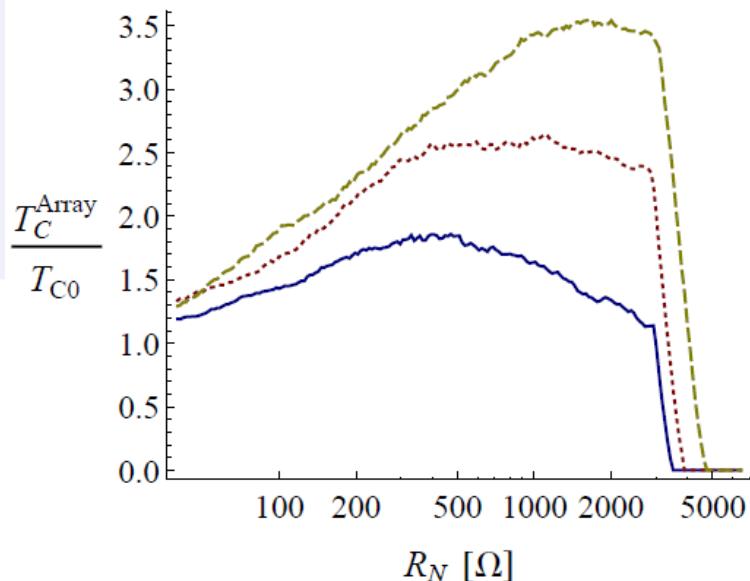
Size

Variance

Packing

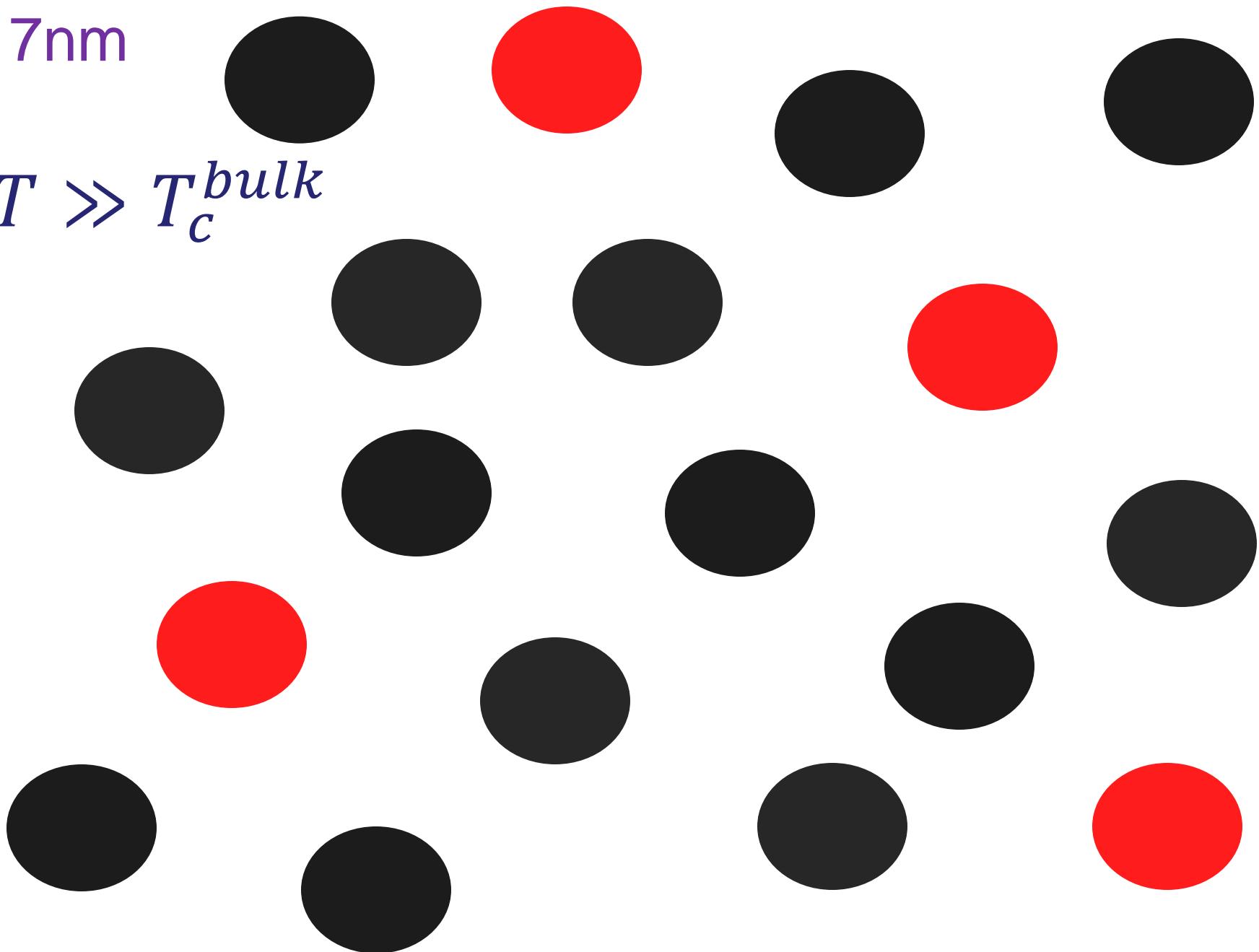


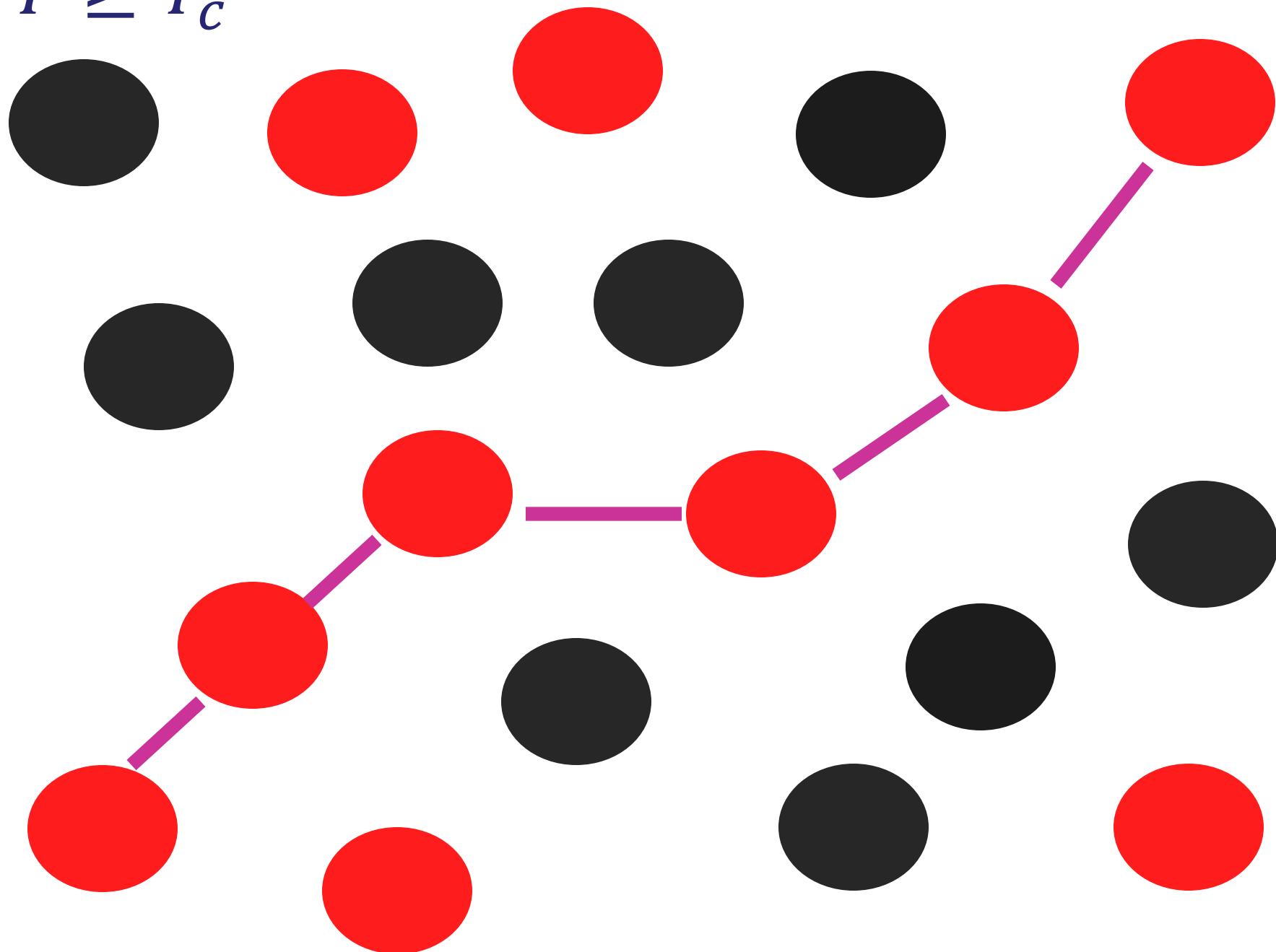
$$T_C = 1.3 T_C^{bulk}$$
$$T_C = 1.5 T_C^{bulk}$$
$$T_C = 3.0 T_C^{bulk} !!!$$

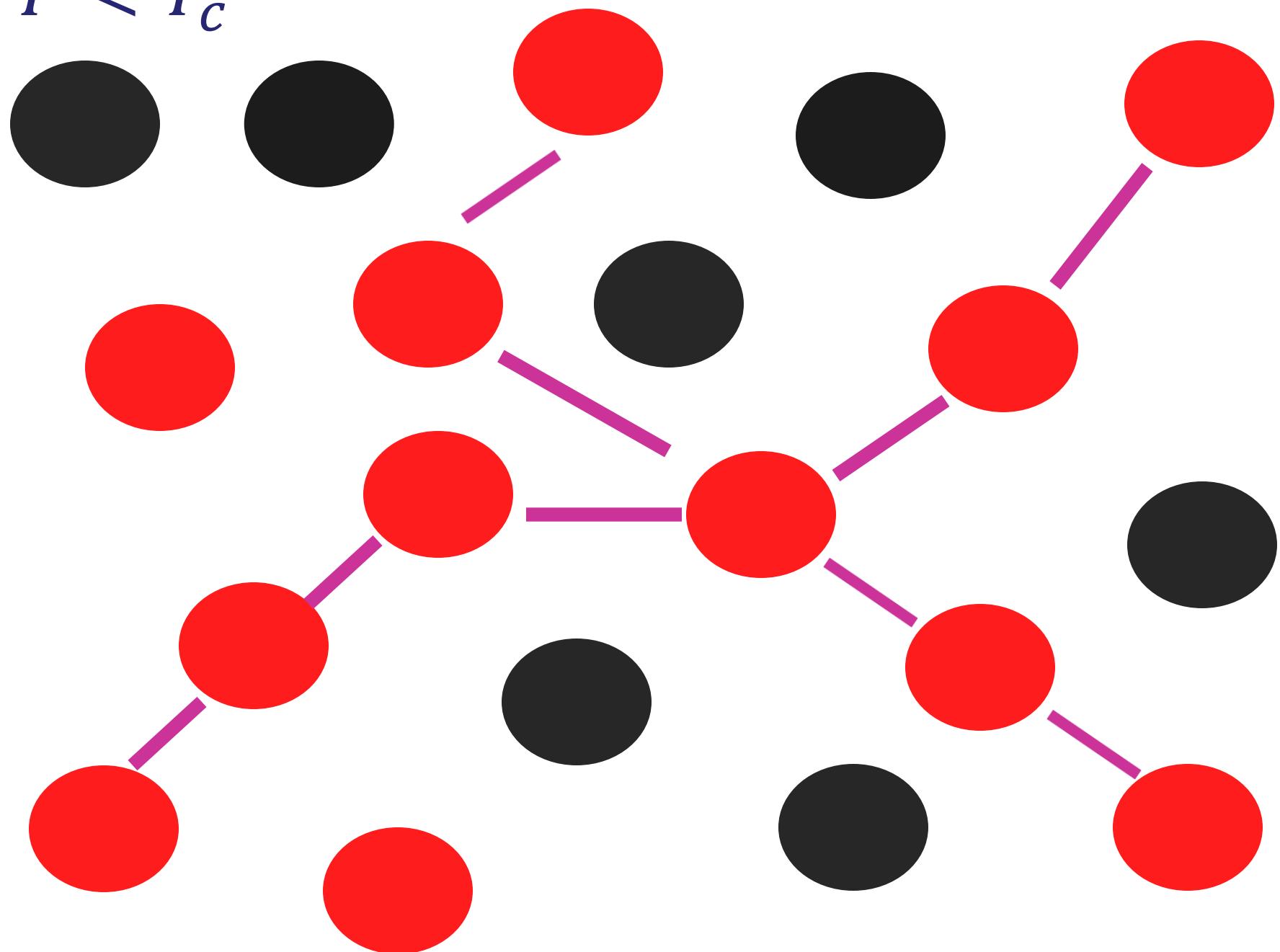


7nm

$T \gg T_c^{bulk}$



$T \geq T_c^{bulk}$ 

$T < T_c^{bulk}$ 

# Designing JJ arrays:

Realistic, doable,  
optimal

3D

Nano  
spheres

Clean

NO BKT

$$P(R) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(R-\bar{R})^2}{2\sigma^2}}$$

Quasi particle  
tunnelling

YES BCS

$$\begin{aligned}\bar{R} &\geq 4nm \\ \sigma &\sim 1nm\end{aligned}$$

Charging

Packing

# How?

Single  
grain

$$\Delta \gg \delta$$

BCS

$$\frac{1}{k_F L} \ll 1$$

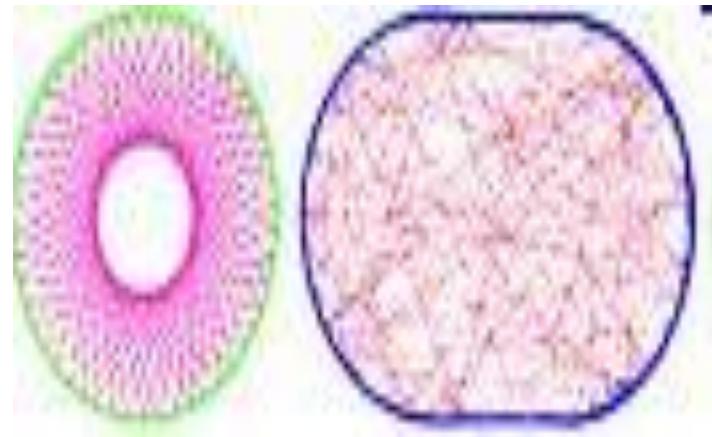
Periodic orbit  
theory

Balian, Bloch, Gutzwiller

Open  
grain

Cutoff long  
orbits

Tunneling



$$\nu(\varepsilon) \Leftrightarrow L_p$$

JJ Array

Mean field

Percolation

# Single grain

Tunneling

Smooth DOS

$$\delta g(\epsilon) = \frac{3}{2} \sqrt{\frac{\pi}{kR}} \sum_{w=1}^{\infty} \sum_{v=2w}^{\infty} (-1)^w \sin(2\theta_{v,w}) \sqrt{\frac{\sin \theta_{vw}}{v}} \sin \Theta_{vw} \omega(R_N, L_P^{v,w}) - \frac{3}{4} \frac{1}{kR} \sum_{w=1}^{\infty} \frac{1}{w} \sin(L_P^w k) \omega(R_N, L_P^w)$$
$$\omega(R_N, L_P) = e^{-\frac{4zL_P R_Q}{R_N \nu(0)v_F h}}$$

$$1 = \frac{\lambda}{2} \int_{-\epsilon_D}^{\epsilon_D} \frac{1}{\sqrt{\epsilon'^2 + \Delta^2}} \frac{\nu(\epsilon')}{\nu_{TF}(0)} \tanh \left( \frac{\beta \sqrt{\epsilon'^2 + \Delta^2}}{2} \right) d\epsilon'$$

Open grain

Weaker size effects

# 3D Array

## Charging

## Hopping

$$S = \frac{1}{2} \int_0^\beta d\tau \sum_i \frac{\dot{\phi}_i^2}{E_Q} - \frac{1}{2} \sum_{\langle ij \rangle} \int_0^\beta d\tau J_{ij} \cos(2(\phi_i(\tau) - \phi_j(\tau))) +$$

Schoen,  
Zaikin,Fazio.

## Quasiparticles

$$2 \sum_{\langle ij \rangle} \int_0^\beta d\tau \int_0^\beta d\tau' G_{ij}(\tau - \tau') \sin^2\left(\frac{1}{4}(\delta\phi_{ij}(\tau) - \delta\phi_{ij}(\tau'))\right)$$

$$J_{ij} = \frac{\Delta_i \Delta_j}{\beta} \frac{R_Q}{R_N} \sum_{l=-\infty}^{\infty} \frac{1}{\sqrt{\left(\left(\frac{\pi(2l+1)}{\beta}\right)^2 + \Delta_i^2\right)\left(\left(\frac{\pi(2l+1)}{\beta}\right)^2 + \Delta_j^2\right)}}$$

HOMOGENEOUS



$$\bar{z} = zp$$

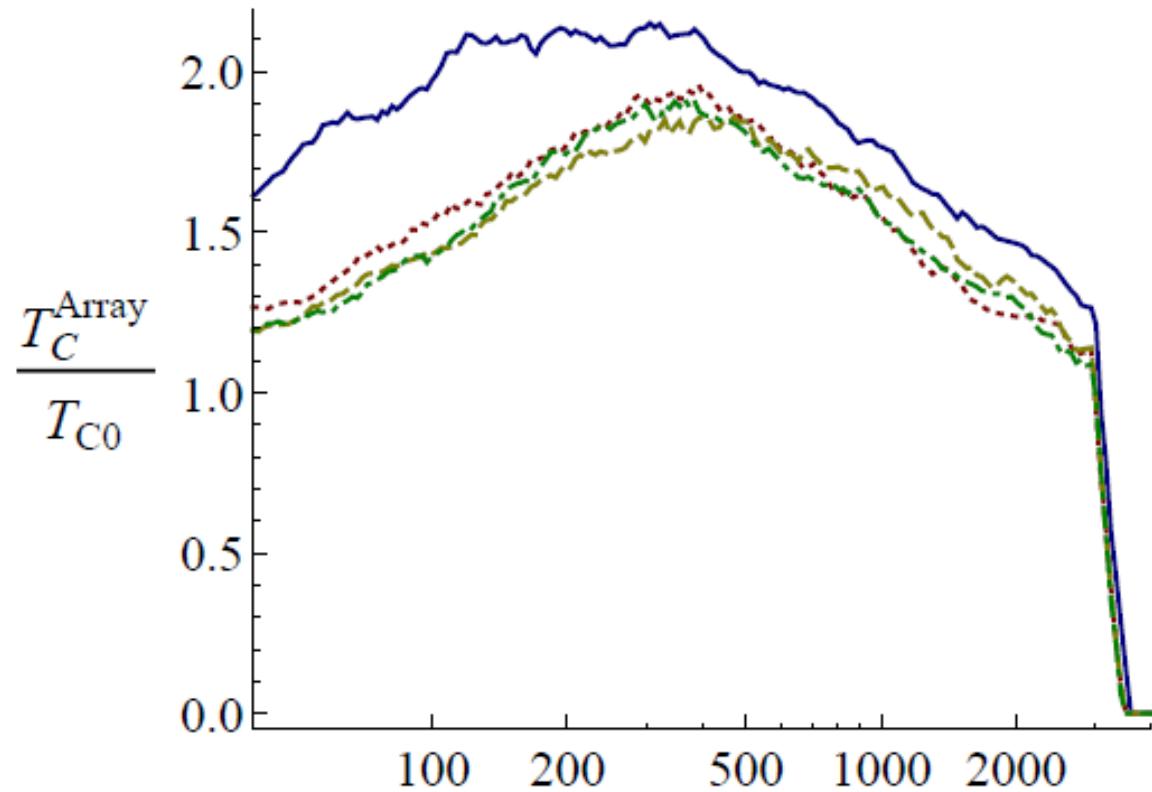
$$1 = \frac{\tilde{E}_Q}{\bar{z}J} + e^{-\beta \tilde{E}_Q/2}$$

Percolation ?

$$\tilde{E}_Q = \left( \frac{1}{E_Q} + \frac{\eta}{E_Q^*} \right)^{-1} \quad J = \frac{\bar{\Delta} R_Q}{2 R_N} \tanh\left(\frac{\beta \bar{\Delta}}{2}\right) \quad E_Q^* = \frac{124e^2 \bar{\Delta} R_N}{3\pi\hbar}$$

$$\sigma = 0.1, 0.6, 1, 1.4 \text{ nm}$$

$$\bar{R} = 5\text{nm} \quad \lambda = 0.25$$



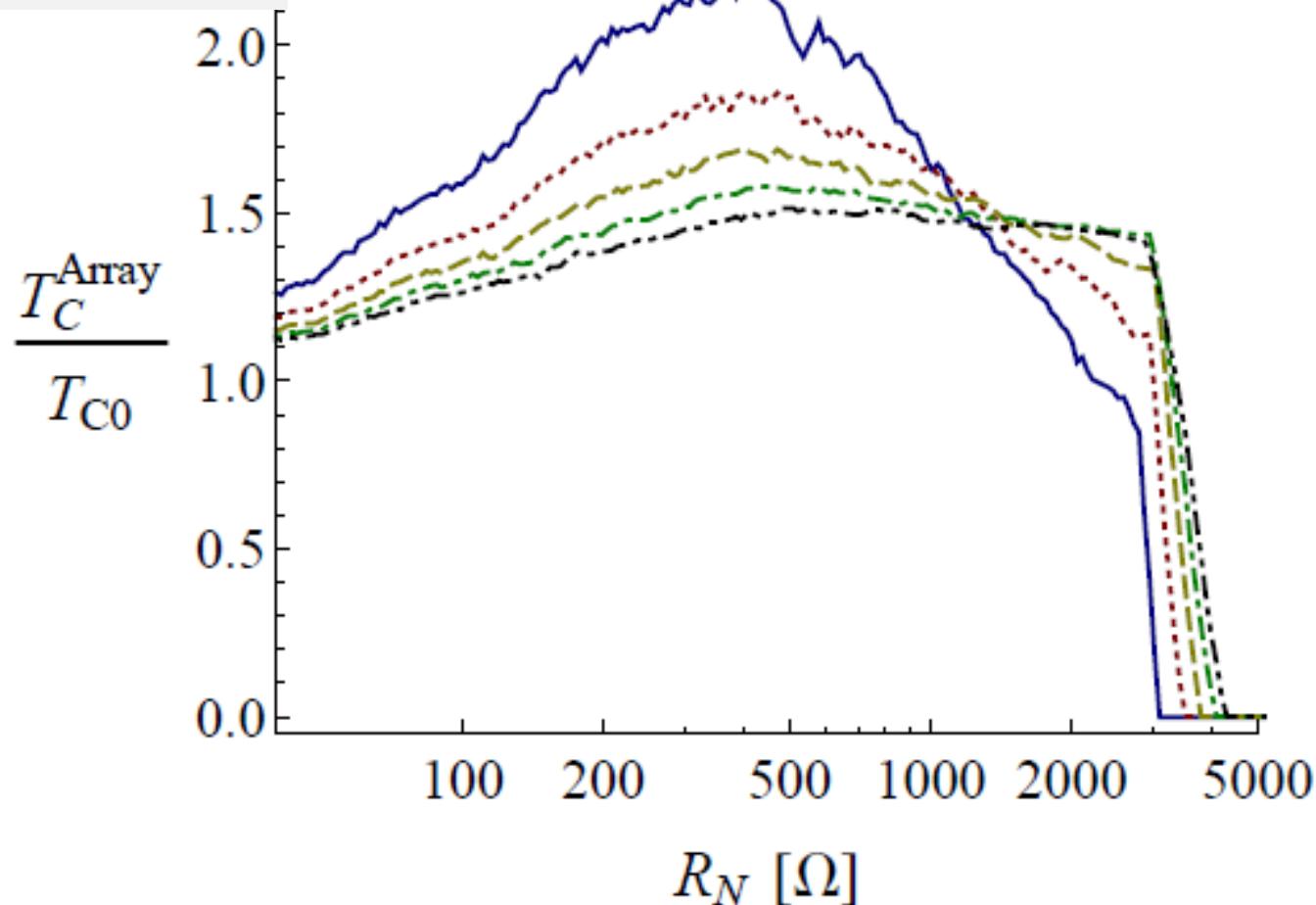
$$P(R) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(R-\bar{R})^2}{2\sigma^2}}$$

$$R_N [\Omega]$$

$$\lambda = 0.2, 0.25, 0.3, 0.35$$

$$\sigma = 1 \text{ nm}$$

$$\bar{R} = 5 \text{ nm}$$



*Packing = Cubic, BCC, FCC*

$$\sigma = 1 \text{ nm}$$

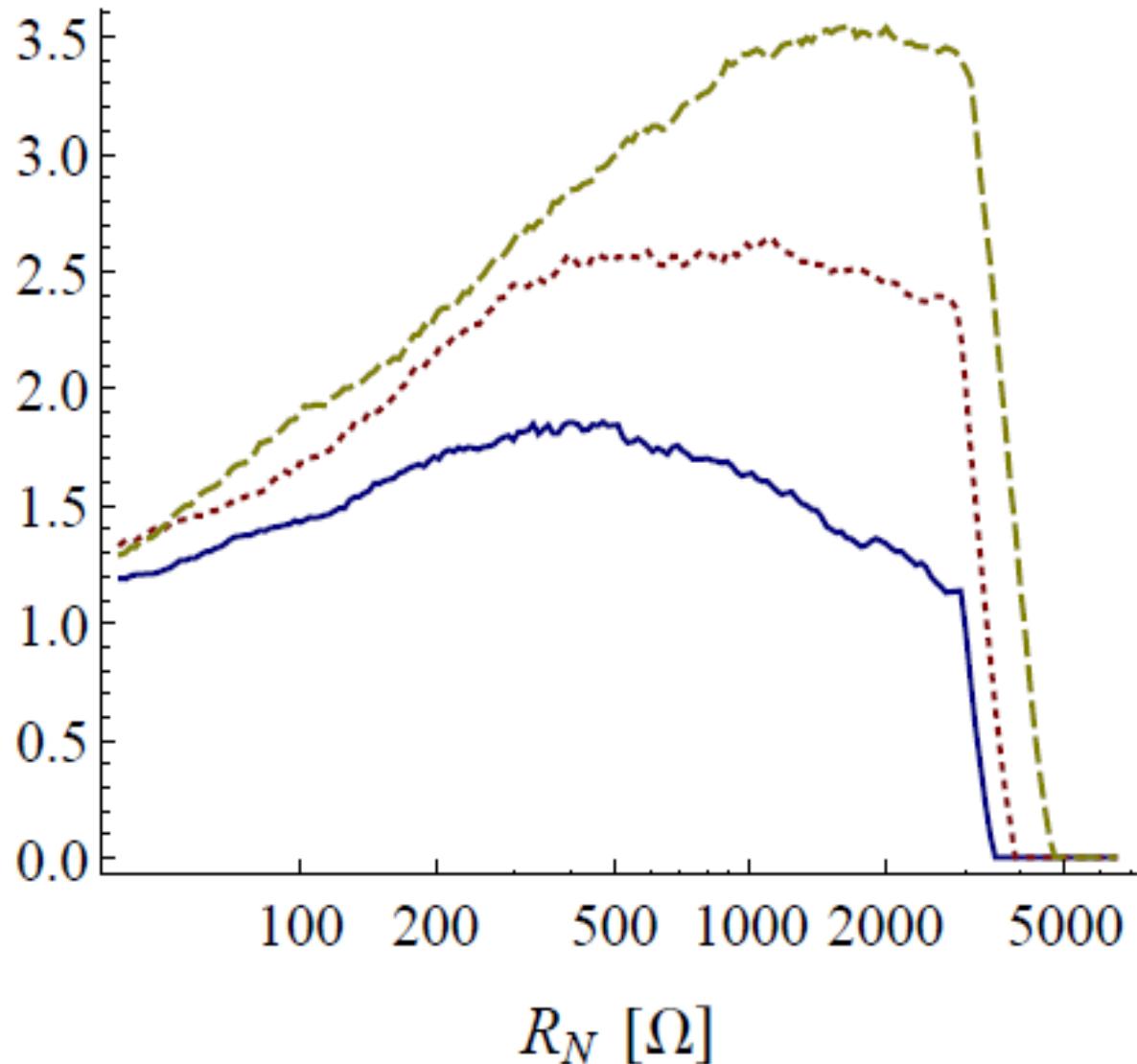
$$\bar{R} = 5 \text{ nm}$$

$$\lambda = 0.25$$

$$\frac{T_C^{\text{Array}}}{T_{C0}}$$

Enhancement  
is possible!

Experiments?



# Origin

Percolation?

$T_c$

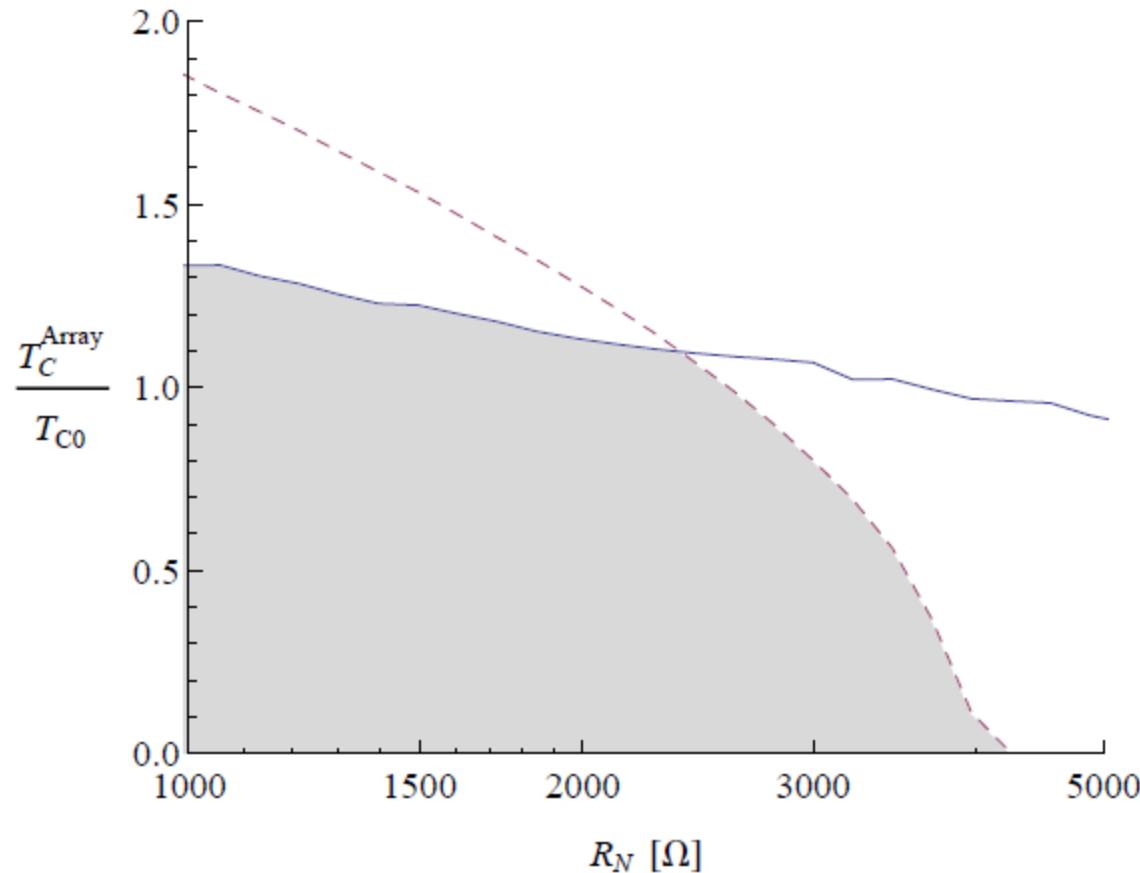
Phase dynamics?

$\uparrow T$   
 $\downarrow \#SCG$

# Percolation

# ??

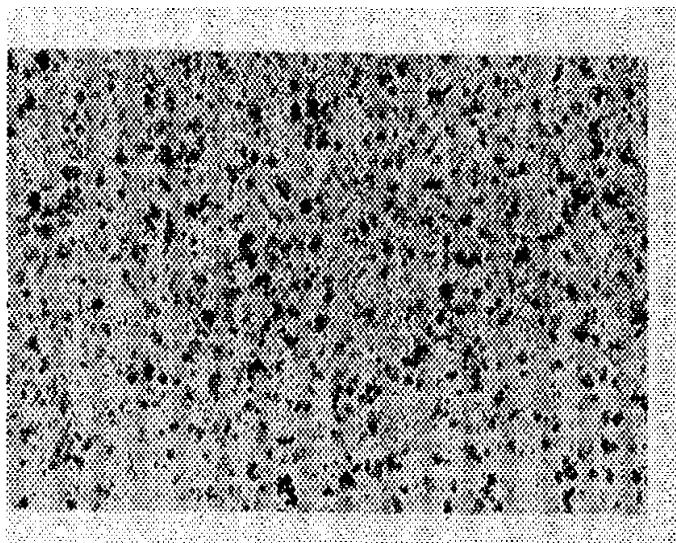
# Phase fluctuations



$R=5\text{nm}$   $\sigma=1\text{nm}$   $\lambda=0.3$

# Experiments:

1960

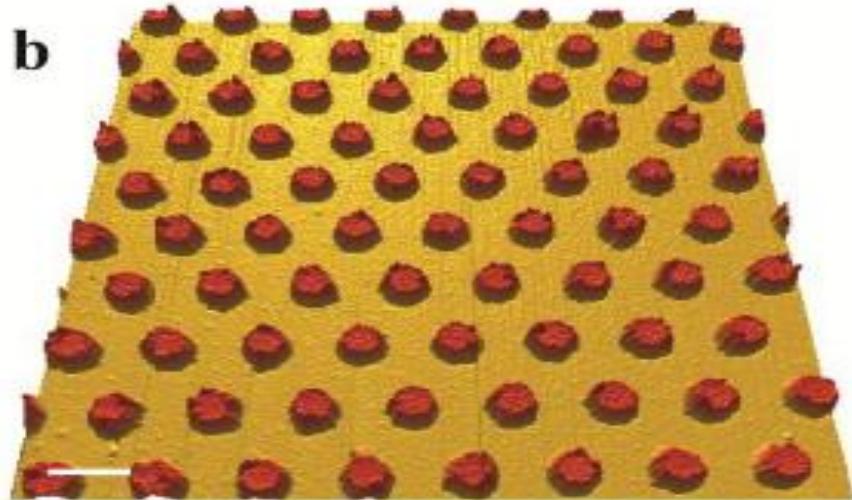


Abeles, Cohen, Cullen, PRL17, 632 (1966)

$L \sim 5\text{nm}$

No Control

2012



Control

$L \sim 50\text{nm}$

$L \sim 5\text{nm}?$

Thanks!